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Observations of geographically correlated orbit errors for TOPEX/Poseidon using the global positioning system

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Abstract. We have compared Global Positioning System (GPS)-based dynamic and reduced-dynamic TOPEX/Poseidon orbits over three 10-day repeat cycles of the groundtrack. The results suggest that the prelaunch joint gravity model (JGM-1) introduces geographically correlated errors (GCEs) which have a strong meridional dependence. The global distribution and magnitude of these GCEs are consistent with a prelaunch covariance analysis, with estimated and predicted global rms error statistics of 2.3 and 2.4 cm rms, respectively. Repeating the analysis with the post-launch joint gravity model (JGM-2) suggests that a portion of the meridional dependence observed in JGM-1 still remains, with a global rms error of 1.2 cm.

Introduction

Our knowledge of the radial component of the orbit, as obtained through the process of precision orbit determination (POD), is of fundamental importance to TOPEX/Poseidon. Owing to the relevance of POD to the mission, three precision tracking systems were adopted: Satellite Laser Ranging (SLR), Doppler Orbitography and Radio-positioning Integrated by Satellite (DORIS), and the Global Positioning System (GPS). Since the launch of TOPEX/Poseidon on August 10, 1992, a number of studies have been conducted to assess the performance of these tracking systems. This paper presents results obtained from an intercomparison of GPS-based dynamic and reduced-dynamic orbits [Yunck et al., 1994; Schutz et al., 1994].

Orbit Errors: Time Series

Figure 1 is a typical time series for the orbital height differences between two dynamic orbits, one produced with GPS data and the other with SLR and DORIS data for repeat cycle 18 (March 10–20, 1993). The JGM-2 gravity model (basically the JGM-1 gravity model with the addition of TOPEX/Poseidon SLR and DORIS data from September 20, 1992 through February 18, 1993 [Nerem et al., 1993(a)]) was used in both cases. The time series can be characterized as a modulated one-cycle-per-revolution (1-cpr) error which has plagued the recovery of sea level from altimetry from previous missions. The one-cycle-per-day (1-cpd) modulation is due to gravity model errors and perhaps the daily updates made to the nongravitational force model parameters during the orbit determination process [Chelton and Schlax, 1993]. The rms value of the radial orbit differences is 2.7 cm, which represents an excellent agreement.

The fact that orbit ephemerides compare well is a necessary, but not sufficient, condition for demonstrating their accuracy. For example, Figure 1 says very little about the accuracy of JGM-2 since the gravity-induced orbit error is common to both ephemerides and hence is unobservable in the orbit differences. However, an assessment of the gravity modeling errors can be made through the comparison of dynamic and reduced-dynamic orbits.

Dynamic orbits, i.e. orbits that depend on dynamic models for propagation of the satellite state, are subject to errors in the initial conditions and the models used to propagate the initial conditions forward in time. Initial condition error arises from tracking data noise, systematic errors in the tracking data, the distribution of tracking data, errors in the orientation and origin of the terrestrial reference frame (station locations), and errors in the dynamic models themselves.

Based on the linear orbit perturbation theory [Kaula, 1966], all terms in the gravity model give rise to perturbations at or near 1 cpr in the radial component of the orbit, which accounts for the broadening of the error spectrum at that frequency [Nerem et al., 1993(b)]. This is also the dominant frequency for nongravitational force model errors; however, the spectral line is much narrower than it is for gravity and, as discussed by Chelton and Schlax [1993], splits into two spectral lines at (1 cpr+1 cpd) and (1 cpr−1 cpd) due to gravity terms of degree m=1 and the estimation of daily nongravitational force parameters.

Kinematic orbits depend only on the tracking metric for propagation of the satellite state, i.e., they do not require dynamic models and are thereby subject only to errors associated with the tracking data. However, a purely kinematic solution requires an extremely robust three-dimensional observing geometry with continuous tracking from multiple stations. Although the fidelity of the GPS observations makes this plausible for TOPEX/Poseidon, a pure kinematic solution is inadvisable owing to limitations of the flight receiver configuration [Melbourne et al., 1994]. Instead, a hybrid of both kinematic and dynamic methodologies has been used, referred to as the reduced-dynamic technique, where small, local geometric corrections are made to a converged dynamic orbit [Yunck et al., 1994]. Insofar that reduced-dynamic GPS orbits have a kinematic component, intercomparisons with GPS, SLR, and DORIS dynamic orbits can reveal deficiencies in the dynamic models and errors in the different tracking systems.

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Such comparisons were done for three TOPEX/Poseidon 10-day repeat cycles covering the periods from March 10–20, June 17–27, and July 17–27, 1993, i.e., cycles 18, 28, and 31, respectively. The gravity model used for the dynamic orbits was JGM-2. It was observed that the differences between the SLR/DORIS dynamic orbits and the GPS reduced-dynamic orbits (3.0–3.4 cm rms) were larger than the differences between the GPS and SLR/DORIS dynamic orbits (2.3–2.7 cm rms). Assuming the reduced-dynamic orbits represent an improvement, this suggests that the dynamic orbits have highly correlated, albeit small, errors that are unobservable in the differences between them. In the next section, we suggest that the error is attributable in part to the gravity models.

Orbit Errors: Geographical

Since the gravity field is fixed in the Earth, orbit errors resulting from the gravity model are expected to be highly correlated with geographical location. Figure 2 shows the transformation of the JGM-1 covariance for the mean of ascending and descending pass errors at any geographical location [cf. Rosborough, 1986], referred to in this paper as the mean GCE. It is important to note that Figure 2 represents the global distribution of the standard deviation for the radial orbit error, i.e., the statistical nature of the error, and therefore has no algebraic sign. The spectral energy of the mean GCE is mostly at low wavenumber; therefore, it is expected that the errors will vary slowly over large expanses of the Earth's surface, as depicted by Figure 2.

Because of the discrete nature of the groundtracks (which have a 314 km equatorial spacing for TOPEX/Poseidon), along with temporal variations in initial condition error and nonconservative force model error, geographical representations of the orbit error are not as well behaved as shown in Figure 2. Since we are interested in separating the mean GCE from these other error sources, it is instructive to review the nature of geographical and temporal orbit errors.

The GCE can be defined as that part of the gravity-induced orbit error that is temporally invariant at a given geographical location. It can exhibit itself in a number of different, albeit related, ways depending on the application of interest. For example, the GCE associated with the mean sea surface derived by smoothing a global altimeter data set that includes ascending and descending passes–the mean GCE – will be manifestly different than it would be for a mean sea surface derived from ascending or descending passes alone. The GCE associated with the differences in sea level measured at points where the ascending and descending passes cross, referred to as crossovers, will be different still. It is beyond the scope of this paper to provide a rigorous explanation for these differences, but we note that they are due to the correlations between the ascending and descending orbit errors.

Geographical projections of actual orbit error, as portrayed in the next set of figures, reveal dominant large scale features with attendant small scale spatial variability. This spatial variability is due to both geographical and temporal orbit error and appears as trackiness in geographical displays of the data. In an attempt to suppress some of this trackiness, we have represented the orbit differences on a $1°\times1°$ grid where the data within a 600 km radius was used to form a weighted average of both ascending and descending data. The weighting factor was chosen to be proportional to the inverse distance from the grid point. We note that some implicit smoothing was accomplished by including the data from three different cycles of the mission, a strategy which reduces that part of trackiness due to temporal variations.

The GCEs in the JGM-1 gravity model are evident in Figure 3a, where the global distribution of the orbital height differences between the GPS reduced-dynamic and GPS dynamic ephemerides using JGM-1 is shown. We note that the reduced-dynamic technique can be somewhat sensitive to the dynamic orbit adopted as a priori, depending on the characteristics chosen for the local geometric corrections. For the level of error represented by JGM-1 and JGM-2, the sensitivity of the reduced-dynamic strategy adopted by Yunck et al., [1994] is almost negligible. Nonetheless, to ensure no preferential weighting of the reduced-dynamic solution to the TOPEX/Poseidon tuned JGM-2 model, JGM-1 was adopted for the a priori dynamic orbit.

To first order, the orbital differences in Figure 3a are geographically correlated and have a strong meridional dependence. This dependence can be approximated by a large-scale positive anomaly in the Indian Ocean and a large-scale negative anomaly in the eastern Pacific Ocean. It was observed that this anomaly is present in each of the maps for cycles 18, 28, and 31; therefore, even though there were slight variations in magnitude, it is not an ephemeral feature. As will be discussed below, a spherical harmonic representation of the data better defines the GCE since it virtually removes the disparity between neighboring orbital tracks resulting from residual trackiness (cf. Figure 3b). The fact that the geographical distribution of these orbit differences so closely corresponds to the expected geographically correlated error pattern for JGM-1 (cf. Figure 2) strongly suggests that it has its origin in the gravity model.

The most compelling evidence that this anomaly is attributable to an error in the JGM-1 gravity model is provided by Figure 4, where the global distribution of the mean orbital height differences between two GPS-based dynamic orbits – one produced with JGM-1 and the other with JGM-2 – is shown. Note that these differences come entirely from the gravity model since this is the only difference between the two cases. This figure is remarkably similar to Figure 3b. Assuming that JGM-2 represents an improvement, these results show that GPS reduced-dynamic orbits have recovered from a significant amount of geographically correlated error introduced by JGM-1. Extending the analysis to include the post-launch gravity model (JGM-2) suggests that a portion of the meridional dependence observed in JGM-1 still remains (cf. Figure 5).

To quantify these conclusions, we have adopted spherical harmonics to discriminate between the mean GCE and other orbit errors (trackiness). A tenth degree and order spherical harmonic representation of the differences between the dynamic and reduced-dynamic orbit heights for the combined set of three cycles was obtained using a least-squares process. It was observed that most of the energy was absorbed by the low order terms ($n < 6$) and that $n = 10$ was adequate for establishing the noise-floor (the point at which little improvement to the fit can be realized by further expansion of the model.) Table 1 summarizes the results obtained from three case studies, where column 2 shows the rms of the prefit orbit difference, column 3 shows the rms of the postfit residuals, and column 4 shows the rms energy contained in the spherical harmonic representation of these

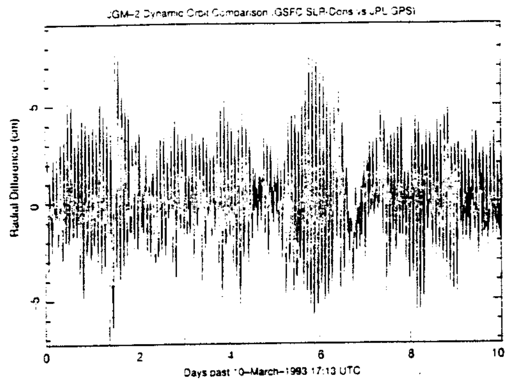

Figure 1. Time series of the differences in the radial component of a dynamic orbit determined with GPS demonstration receiver data and a dynamic orbit determined with SLR and DORIS for cycle 18 of TOPEX/Poseidon, both generated with JGM-2.

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Figure 3b. A tenth order and degree spherical harmonic representation of the data used for Figure 3a (2.3 cm rms).

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Figure 2. Geographically correlated radial orbit error for TOPEX/Poseidon predicted by the JGM-1 error covariance.

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Figure 4. A tenth order and degree spherical harmonic representation of orbital height differences between GPS dynamic orbits generated with the JGM-1 gravity model and GPS dynamic orbits generated with JGM-2 gravity model for cycles 18, 28, and 31 of TOPEX/Poseidon (1.8 cm rms).

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Figure 3a. Global distribution of the orbital height differences between the GPS dynamic and GPS reduced-dynamic ephemerides using JGM-1 for cycles 18, 28, and 31 of TOPEX/Poseidon (3.5 cm rms).

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Figure 5. A tenth order and degree spherical harmonic representation of the orbital height differences between the GPS dynamic and GPS reduced-dynamic ephemerides using JGM-2 for cycles 18, 28, and 31 of TOPEX/Poseidon (1.2 cm rms).

Table 1. TOPEX/Poseidon geographically correlated orbit errors from 10x10 spherical harmonic (SH) expansion of orbit differences. The postfit RMS is the error remaining in the orbit differences after removal of the SH functions. The prelaunch prediction is derived from evaluation of the gravity field covariance matrix (units are cm).

Case	Prefit RMS	Postfit RMS	RMS of mean GCE	Prelaunch prediction of mean GCE
•JGM-1 vs JGM-2	2.1	1.2	1.8	--
•JGM-1 vs Reduced dynamic	3.5	2.7	2.3	2.4
•JGM-2 vs Reduced dynamic	2.5	2.3	1.2	1.6

differences. The residuals reported in Table 1 contain that part of the GCE associated with the ascending and descending tracks that is not observable in the mean GCE. As a result, the rms of the residuals provides an indication of the decorrelation at the crossover points, information that would be lost if the data were averaged prior to the fit. In column 5, we have compiled our estimates of the TOPEX/Poseidon geographically correlated error based on the applications of linear theory to the error covariance matrix.

In Case 1 we consider two dynamic orbits based on JGM-1 and JGM-2. If the GCE for the ascending and descending passes were perfectly correlated with the mean GCE, the rms value of the residuals would be extremely small. This case shows that a 1.8 cm rms mean GCE exists between JGM-1 and JGM-2, with a 1.2 cm rms residual orbit error attributable to inhomogeneities in the GCEs associated with the ascending and descending tracks, as well as trackiness introduced by the different sets of initial conditions. For Case 2 we consider the difference between the JGM-1 dynamic and reduced-dynamic orbits. In this comparison, the rms value of the spherical harmonic surface represents our estimate of the mean GCE introduced by JGM-1 (cf. Figure 3b). The residual rms represents errors associated with the trackiness of the data. In Case 3, we repeat the analysis with JGM-2 and find a mean GCE of 1.2 cm rms (cf. Figure 5b), which is a significant improvement over the 2.3 cm rms found for JGM-1. For JGM-1, our estimate of the mean GCE corresponds almost exactly to the prelaunch prediction of 2.4 cm rms (cf. Figure 2). For JGM-2, the predicted 1.6 cm rms is somewhat higher than the 1.2 cm rms found here, but in any case this suggests that the gravity-induced errors have indeed been reduced to the level of 2 cm rms. These results not only provide a tangible demonstration of the unprecedented strength of the GPS data from TOPEX/Poseidon, they also serve as evidence of the remarkable achievements in the gravity modeling effort [Tapley et al., 1993].

Comments

The results presented here suggest that the GPS reduced-dynamic technique has great potential for reducing the error in the orbit of TOPEX/Poseidon, even in the presence of small dynamic model errors. Though JGM-2 is definitely an improvement over JGM-1, a measurable amount of energy remains in the differences between reduced-dynamic and dynamic orbits determined with JGM-2. It follows that, when used with dynamic orbit determination techniques, GPS data is capable of improving the Earth's gravity model [e.g., Schutz et al, 1994]. Once such an improved model is obtained, the GPS dynamic orbits are expected to be even more accurate for TOPEX/Poseidon. For low Earth orbiters, however, both gravitational and nongravitational forces are much more difficult to model. Therefore, the GPS reduced-dynamic technique will be particularly important for low-altitude altimeter missions where errors due to atmospheric drag and gravity are especially large.

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