

**COMPARISON OF TOPEX/POSEIDON ORBIT DETERMINATION
SOLUTIONS OBTAINED BY THE GODDARD SPACE FLIGHT
CENTER FLIGHT DYNAMICS DIVISION AND PRECISION ORBIT
DETERMINATION TEAMS***

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Orbit determination results are obtained by the Goddard Space Flight Center (GSFC) Flight Dynamics Division (FDD) using the Goddard Trajectory Determination System (GTDS) and a real-time extended Kalman filter estimation system to process Tracking Data and Relay Satellite (TDRS) System (TDRSS) measurements in support of the Ocean Topography Experiment (TOPEX)/Poseidon spacecraft navigation and health and safety operations. GTDS¹ is the operational orbit determination system used by the FDD, and the extended Kalman filter was implemented in an analysis prototype system, the Real-Time Orbit Determination System/Enhanced (RTOD/E)². The Precision Orbit Determination (POD) team within the GSFC Space Geodesy Branch generates an independent set of high-accuracy trajectories to support the TOPEX/Poseidon scientific data. These latter solutions use the Geodynamics (GEODYN) orbit determination system with laser ranging tracking data.

The TOPEX/Poseidon trajectories were estimated for the October 22–November 1, 1992, timeframe, for which the latest preliminary POD results were available. Independent assessments were made of the consistencies of solutions produced by the batch and sequential methods. The batch cases were assessed using overlap comparisons, while the sequential cases were assessed with covariances and the first measurement residuals. The batch least-squares and forward-filtered RTOD/E orbit solutions were compared with the definitive POD orbit solutions. The solution differences were generally less than 10 meters (m) for the batch least squares and less than 18 m for the sequential estimation solutions. The differences among the POD, GTDS, and RTOD/E solutions can be traced to differences in modeling and tracking data types, which are being analyzed in detail.

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INTRODUCTION

This paper assesses the Ocean Topography Experiment (TOPEX)/Poseidon orbit determination accuracy of the Tracking and Data Relay Satellite (TDRS) System (TDRSS)-based orbit solutions using an operational batch least-squares system and a prototype sequential orbit determination system at Goddard Space Flight Center (GSFC) Flight Dynamics Division (FDD). The TDRSS-based orbit solutions are compared with the preliminary high-precision orbit solutions obtained by the GSFC Space Geodesy branch using laser tracking measurements.

The National Aeronautics and Space Administration (NASA) has completed a transition from tracking and communications support of low Earth-orbiting satellites with a ground-based station network, the Ground Spaceflight Tracking and Data Network (GSTDN), to the geosynchronous relay satellite network, the TDRSS. TDRSS consists of four operational geosynchronous spacecraft and the White Sands Ground Terminal (WSGT) at White Sands, New Mexico. The ground network provided only about 15 percent visibility coverage, while TDRSS can provide 85 percent to 100 percent coverage, depending on spacecraft altitude.

The Bilateral Ranging Transponder System (BRTS) provides range and Doppler measurements for determining each TDRS orbit. The ground-based BRTS transponders are tracked as if they were TDRSS-user spacecraft. Since the positions of the BRTS transponders are known, their ranging data can be used to precisely determine the trajectory of the TDRSSs.

The accuracy requirements on the Space Geodesy Branch Geodynamics (GEODYN) orbit determination solutions, used to analyze the sea surface height measurements obtained by the TOPEX/Poseidon radar altimeter, are extremely stringent. The definitive orbit determination requirements for the TOPEX/Poseidon mission science data include a maximum 13-centimeter (cm) radial position error. The accuracy of the POEs is being verified through the use of the TOPEX/Poseidon science data. Global radar altimeter measurements of the ocean surface are taken, and then compared with coincident definitive TOPEX ephemerides generated using the ground-based laser tracking. The GEODYN force modeling is then calibrated to minimize the differences between the definitive TOPEX ephemerides and the radar altimeter measurements.

Preliminary high-accuracy ephemerides, with an accuracy slightly worse than 13 cm, will be used to assess the accuracy of FDD-generated orbit determination solutions. The availability of the orbit determination solutions generated by the Space Geodesy Branch provides a unique opportunity to evaluate the accuracy of the orbit determination systems used by the FDD for operational and analysis navigation support.

This paper assesses the orbit determination accuracy of the batch least-squares method, which is used for operational orbit determination support. The paper also assesses the accuracy of a sequential method implemented in a prototype system, used for analysis in the GSFC Flight Dynamics Facility (FDF). The batch weighted least-squares algorithm implemented in the Goddard Trajectory Determination System (GTDS) estimates sets of orbital elements, force modeling parameters, and measurement-related parameters that minimize the squared difference between observed and calculated values of selected tracking data over a solution arc.¹ GTDS operates on the mainframe computer system at the FDF.

The sequential estimation algorithm implemented in a prototype system, the Real-Time Orbit Determination/Enhanced (RTOD/E), simultaneously estimates the TDRSS user and relay spacecraft orbital elements and other parameters in the force and observation models at each measurement time.^{2,5} RTOD/E performs forward filtering of tracking measurements using the extended Kalman filter with a process noise model to account for serially correlated, geopotentially induced errors, as well as Gauss-Markov processes for drag, solar radiation pressure, and measurement biases. The main features of RTOD/E can be found in an earlier paper.³

The estimated TOPEX/Poseidon ephemerides were obtained for October 22–November 1, 1992. This timeframe was chosen because it was the latest for which the preliminary Precision Orbit Determination (POD) results were then available. Independent assessments were made to examine the internal consistencies of results obtained by the batch and sequential methods.

This paper describes the orbit determination and evaluation procedures used in this study, summarizes POD solutions,⁴ describes the results obtained by the batch least-squares and sequential estimation methods, provides the resulting consistency and cross comparisons, and presents the conclusions of this study.

ANALYSIS PROCEDURES

This section describes the analysis procedures used in this study and provides a description of the tracking measurements and orbit determination and modeling methods.

Tracking Measurements

The TOPEX/Poseidon spacecraft was launched aboard an Ariane 42P expendable launch vehicle in August 1992. In October 1992, maneuvers were completed that moved the spacecraft into its operational orbit, which is circular with an inclination of 66°, an altitude of 1336 kilometers (km), a period of 112 minutes (min), and a 10-day repeat period ground track. The time period chosen for this study was 19:33 hours (hr) coordinated universal time (UTC) on October 22, 1992, through 21:30 hr UTC on November 1, 1992, which corresponds to the fourth 10-day ground track repetition, hereafter referred to as *Cycle 4*.

Tracking measurements from the TDRSS, used for TOPEX/Poseidon operational orbit navigation support by the FDF, were used to generate the GTDS and RTOD/E ephemerides. The GTDS and RTOD/E orbit solutions were obtained using one-way and two-way Doppler data.

During Cycle 4, there were 3 TDRSSs actively tracking user spacecraft; however, at any given time only two TDRSSs tracked TOPEX. The three active TDRSSs were TDRS-West (TDRS-5, 174 deg. west longitude), TDRS-East (TDRS-4, 41 deg. west longitude), and TDRS-Spare (TDRS-3, 62 deg. west longitude). TDRS-1 was not tracking user spacecraft.

The tracking consisted of an average of 10 passes of one-way Doppler observations and 11 passes of two-way Doppler observations per day, with the average pass lasting 40 min. A representative daily TDRSS tracking data distribution from Cycle 4 is shown in Fig. 1. Passes labeled "2" consist of two-way Doppler observations, while passes labeled "1" consist of one-way Doppler observations. BRTS tracking coverage of each TDRS spacecraft typically consists of twelve to fifteen 5-min passes per day.

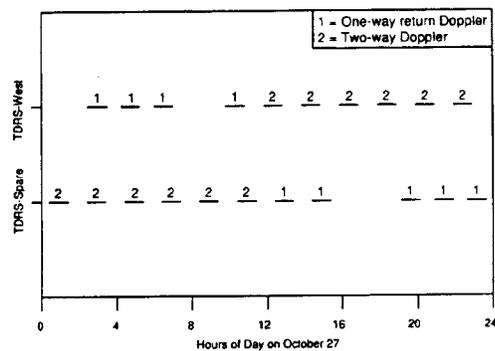


Fig. 1. TDRSS Tracking Data for TOPEX

The POD team used ground-based laser ranging data to generate the precision orbit ephemerides (POEs). The laser tracking data network consists of approximately 50 ground stations located around the world. Fifteen of these stations are specifically designated to support TOPEX/Poseidon tracking. Most of the stations are located in the United States, Europe, and Australia. During Cycle 4, 154 passes of laser range data, from 19 ground stations, were distributed over the 10-day period. Each pass lasted from 10 to 15 min. Table 1 shows the number of laser tracking data passes and observations for each day in Cycle 4.

Table 1

NUMBERS OF LASER TRACKING DATA PASSES AND OBSERVATIONS FOR EACH DAY IN CYCLE 4

Day	Passes	Observations
10/22/92	3	55
10/23/92	20	501
10/24/92	15	247
10/25/92	25	487
10/26/92	14	233
10/27/92	15	324

Day	Passes	Observations
10/28/92	18	286
10/29/92	11	249
10/30/92	15	247
10/31/93	10	184
11/01/92	8	184

Orbit Determination Methods and Modeling

This section describes the orbit determination methods and the modeling used to generate the TOPEX/Poseidon solutions and ephemerides, and provides the orbit determination methods and modeling for the POEs, GTDS batch least-squares solutions, and RTOD/E sequential estimation solutions.

Precision Orbit Ephemerides. The POEs are generated by Space Geodesy Branch POD team personnel using the GEODYN program. Each POE spans a 10-day period coincident with a project-defined beginning and end of a repeatable ground track. GEODYN, like GTDS, uses a batch least-squares estimation process to fit the laser tracking data and estimate a solution.

At the time of this writing, the POD team was analyzing and improving the accuracy of the POEs and had not finalized the GEODYN force modeling. Therefore, the Cycle 4 POE used in this study is preliminary and does not represent the quality of the final POEs to be used to support the TOPEX/Poseidon science data. The quality of the preliminary POEs is discussed below.

The important force models and parameters used in the preliminary Cycle 4 POE are given in Table 2. The TOPEX/Poseidon solve-for parameters consist of the spacecraft state vector, 10 drag coefficients (one per day), the coefficient of solar radiation pressure, and a 127 once-per-revolution along-track accelerations. This once-per-revolution along-track acceleration was introduced to better model an anomalous spacecraft body-fixed acceleration discovered shortly after launch. The Cycle 4 POE was generated using 116 out of 154 available laser tracking passes.

Batch Least-Squares Estimation. The batch least-squares estimation algorithm used by GTDS for this analysis is the same as that used for operational orbit determination and navigation support of the TOPEX/Poseidon mission by the GSFC FDF. The procedure for operational support includes solving for the spacecraft state, onboard ultra-stable oscillator (USO) bias and drift parameters, and an along-track thrust estimation parameter using two-way and one-way Doppler measurements only. Range measurements were excluded from the solutions to avoid software limitations in solving for uncorrected biases, which have been found to reduce the orbit solution quality. The modeling and state solve-for parameters used for this analysis have been enhanced to provide more accurate results and to take advantage of modeling and techniques not currently in operational use. Specifically, the state space was expanded to include estimation of the coefficient of solar radiation pressure and nominally daily along-track thrust parameters that were intended to compensate for the effects of an unmodeled, variable, and primarily body-fixed force acting on the spacecraft that has been observed in the operational support. The modeling and options used are presented in Table 3.

The TDRS orbits used to process the one-way return and two-way TDRSS Doppler data used by the batch estimation were obtained from the operational orbit solutions for TDRS-Spare and TDRS-West that were determined separately using only BRTS tracking. During the study period, TOPEX was tracked by two TDRSs: TDRS-Spare and TDRS-West. On November 2, 1992, TDRS-East replaced TDRS-Spare for TOPEX support affecting one of the GTDS solutions. The TDRS orbits are maintained on a database of nominally

Table 2
FORCE MODELING AND PARAMETERS USED IN THE CYCLE PRELIMINARY 4 POE

Orbit Determination Parameter or Option	POE Values
Estimated parameters	Orbital state, drag coefficient (C_D , one per day), coefficient of solar radiation pressure (C_R), along-track acceleration
Integration type	11th order fixed-step Cowell
Coordinate system of integration	True-of-reference
Integration step size	30.0 sec
Tracking data	Ground-based laser ranging data
Data rate	1 per 30 sec
DC convergence parameter	2 percent between iterations
Editing criterion	3.5 σ
Satellite area model	Box/wing model
Satellite mass	2417.163 kg
Geopotential model	70 x 70 JGM*-1
Atmospheric density model	Drag temperature model
Solar and lunar ephemerides	DE-200
Tropospheric refraction correction	Yes
Polar motion correction	Yes
Solid Earth tides	Yes
Ocean tides	Yes
Plate motion	Yes
Earth radiation pressure	Yes

*JGM = Joint Gravity Model

Table 3
PARAMETERS AND OPTIONS USED IN THE GTDS SOLUTIONS (1 of 2)

Orbit Determination Parameter or Option	GTDS Values
Estimated parameters	Orbital state, thrust coefficients (τ , one/day), coefficient of solar radiation pressure (C_R), ultra-stable oscillator bias and drift
Integration type	Cowell 12th order
Coordinate system of integration	Mean-of-J2000
Integration step size (seconds)	60 sec
Tracking measurements	TDRSS two-way Doppler TDRSS one-way return Doppler
Data span	6 Days 16 hr—two resonance potential beat periods
Data rate	1/40 sec
DC convergence parameter	.00005
Editing criterion	3- σ Central angle of 79.48 deg
Measurement weight sigmas	.25 Hz two-way, .13 Hz one-way
Satellite area model	Variable mean area model
Satellite mass	2417.2 kg
Geopotential mode	50 x 50 GEM*-T3

Table 3

PARAMETERS AND OPTIONS USED IN THE GTDS SOLUTIONS (2 of 2)

Orbit Determination Parameter or Option	GTDS Value
Atmospheric density model	Jacchia-Roberts
Solar and lunar ephemerides	DE-200
Coefficient of drag (C_D)	2.3 applied
Ionospheric refraction correction	
Ground-to-spacecraft	Yes
Spacecraft-to-spacecraft	No (central angle edit instead)
User-spacecraft antenna offset correction	Yes
Tropospheric refraction correction	Yes
Polar motion correction	Yes
Solid Earth tides	Yes
Ocean tides	No
Plate motion	No
Earth radiation pressure	No

*GEM = Goddard Earth Model

34-hr-long periods, and are accessed by GTDS to obtain relay orbits at times applicable to the observations. In general, TDRS stationkeeping maneuvers are handled automatically by GTDS when using this system. Occasionally the shorter arc lengths required by the maneuver support result in periods of reduced definitive ephemeris quality. Consequently, some measurements surrounding TDRS maneuvers were edited. In all situations where tracking was reduced to a single TDRS, the data acceptance rates were doubled to keep the distribution and weighting of data as even as possible. Table 4 lists the epochs and data spans used, and Table 5 lists the TDRS maneuvers and spacecraft events occurring during the solution arcs.

Analysis of the operational TOPEX/Poseidon orbit solutions has indicated the presence of an unmodeled spacecraft body-fixed force with a day-to-day variability. Analysis performed by Jet Propulsion Laboratory (JPL) has indicated that the unmodeled force is dependent on the angle between the orbit plane and the Sun.⁷

Table 4

SOLUTION EPOCHS AND DATA SPANS

Epoch	Data Span
10/19/92 19:13	10/19/92 19:13-10/26/92 05:00
10/22/92 20:10	10/22/92 20:10-10/29/92 08:00
10/25/92 16:00	10/25/92 16:00-11/01/92 08:00
10/28/92 16:00	10/28/92 16:00-11/04/92 08:00

Table 5

TDRS AND TOPEX EVENTS DURING THE ENHANCED GTDS SOLUTIONS

Time of Event	Event
10/19/92 19:13	TOPEX steering mode changed from fixed to sinusoidal
10/22/92 20:10	TOPEX solar array panel pitch changed
11/02/92 13:00	TDRS-4 replaces TDRS-3 for TOPEX support (affects 10/28 solution)
11/04/92 00:50	TDRS-East east/west maneuver (affects 10/28 solution)

This force has been observed in addition to what was apparently outgassing during the mission assessment phase. Consequently, in addition to an applied drag force, a series of thrust correction factors, a 1-micronewton continuous alongtrack thrust was estimated. Distribution of the thrust correction factors was nominally daily, with exceptions made for changes in the spacecraft solar array configuration and attitude flight mode. Also, the thrust solve-for parameter spans at the ends of the data arcs were typically 32 hr long; the extra duration was needed to maintain the accuracy of the solve-for parameter. Improved solution overlap values resulted from maintaining the accuracy of the solve-for thrust values at the ends of the solutions.

The enhanced GTDS solutions were evaluated on the basis of a series of overlapping 6.6-day solutions, one every 3 days with epochs 3 days apart, resulting in a nominal 3.6-day overlap. The epochs were placed at the start of the data arcs, and the definitive ephemeris overlap position comparison with the previous solution was used to judge the solution-to-solution consistency, not the absolute accuracy. The tracking data residual statistics and comparison of corresponding solution solve-for parameters were also used to evaluate the enhanced GTDS solutions.

Sequential Estimation. In this work, RTOD/E serves as a research tool for assessing sequentially estimated orbit solutions generated within a realistic FDF environment. RTOD/E execution has been in progress since TOPEX was launched in August 1992. During some portions of the 4 months preceding the analysis, RTOD/E was in a real-time or near-real-time operating mode. At various points, execution was suspended to accommodate maneuvers and adjust tuning parameters. In addition, complete reinitialization of RTOD/E was necessary on two occasions—the first to accommodate a TDRS reassignment, and the second to incorporate USO-stabilized one-way Doppler tracking measurements. Thus, the RTOD/E configuration for the Cycle 4 time period does not apply to all of the previous processing. Tables 6 and 7 provide a detailed description of the models and options used for Cycle 4 RTOD/E processing. The RTOD/E results reflect historical values of daily and 81-day average solar flux and geomagnetic data. The RTOD/E solution state included orbital elements for TOPEX, TDRS-Spare, and TDRS-West. Other estimated quantities included a coefficient of atmospheric drag for TOPEX, a coefficient of solar radiation pressure for each of the three satellites, Doppler measurement biases, and USO bias. The full RTOD/E state error covariance matrix had a dimension of 27 by 27 when not processing BRTS measurements. During BRTS passes, the measurement biases for BRTS range and range difference measurements are added to the state space.

A comparison between RTOD/E and POD ephemerides, resolved in orbit-plane principal directions, provided the primary means of gauging sequential orbit determination accuracy of this analysis. The comparisons were performed in the J2000.0 true-of-date (TOD) coordinate frame. Other indicators of RTOD/E solution quality were provided by the diagonal elements of the state error covariance matrix, integrity of the drag coefficient estimates, and the relationship of the first predicted residual to the residual standard deviation for each tracking pass.

RESULTS AND DISCUSSION

This section presents the TOPEX/Poseidon accuracy assessment analysis results, an assessment of the consistency of the TOPEX/Poseidon ephemerides, and the ephemeris comparison results.

Summary Results of POEs

To assess the quality of the preliminary POEs, overlap comparisons were performed by the POD team for Cycles 3 and 4. The Cycle 3 POE is 10 days in length and spans 23:50 hr UTC on October 12, 1992, through 23:30 hr UTC on October 22, 1992. A special 10-day ephemeris, called an *overlap ephemeris*, was generated by the POD team and overlaps the last 5 days of the Cycle 3 POE and the first 5 days of the Cycle 4 POE.

The root-mean-square (RMS) position differences in the overlap region for Cycle 3, or overlap 1, are 15 cm, 10 cm, and 39 cm in the radial, cross-track, and along-track directions, respectively. In overlap 2, the RMS position differences are 7 cm, 8 cm, and 15 cm in the radial, cross-track, and along-track directions, respectively. These results indicate a high degree of consistency for Cycle 4. The larger differences in Cycle 3 are attributed to the large number of unmodeled spacecraft attitude events occurring during this time.

Table 6

PARAMETERS AND OPTIONS FOR SIMULTANEOUS TOPEX AND TDRS SOLUTIONS

Orbit Determination Parameter or Option	RTOD/E Values	
	TOPEX	TDRS-West/Spare
Estimated parameters	Orbital state, coefficients of drag and solar radiation pressure, TDRSS Doppler tracking measurement biases, USO bias	Orbital state, coefficient of solar radiation pressure, BRTS range and Doppler tracking measurement biases
Integration type	Variation of parameters	Variation of parameters
Coordinate system of integration	Mean of 1950.0	Mean of 1950.0
Integration step size	60 sec	600 sec
Tracking data	TDRSS two-way Doppler and TDRSS one-way return Doppler	BRTS range and range difference
Data rate	One per 30 sec	One per 30 sec
Editing criterion	3- σ	3- σ
Gravity error auto correlation values	2.828 min (Radial) 0.001 min (Along-track) 5.611 min (Cross-track) Errors of omission and commission	N/A
Measurement weight sigmas: Range Doppler	N/A 0.01 Hz	0.50 m 0.01 Hz
Gauss-Markov parameters: Drag half-life Drag sigma C_R half-life C_R sigma Range bias half-life Range bias sigma Doppler bias half-life Doppler bias sigma	840.0 min 0.400 1440.0 min 0.200 N/A N/A 8 min 0.034 Hz	N/A N/A 11520.0 min 0.200 60.0 min 7.0 m 60 min 0.030 Hz
USO fractional noise standard deviation	5×10^{-10}	N/A
USO doweighting standard deviation	10^{-13}	N/A
USO doweighting time constant	10.0 seconds	N/A
GM standard deviation	$0.005 \text{ km}^3/\text{sec}^2$	$0.005 \text{ km}^3/\text{sec}^2$
Satellite area	32 m ²	40 m ²
Satellite mass	2417.200 kg	1824.979, east 1982.022, west

Table 7

FORCE AND MEASUREMENT MODEL SPECIFICATIONS (1 of 2)

Model or Option	RTOD/E Values	
	TOPEX	TDRS-West/Spare
Geopotential model	GEM-T3 (50 x 50)	GEM-T3 (8 x 8)
Atmospheric density model	CIRA 72	N/A
Solar and lunar ephemerides	Analytic	Analytic

Table 7
FORCE AND MEASUREMENT MODEL SPECIFICATIONS (2 of 2)

Model or Option	RTOD/E Values	
	TOPEX	TDRS-West/Spare
Coefficient of drag	Estimated with a priori value of 1.25	N/A
Coefficient of solar radiation pressure	Estimated with a priori value of 1.4	Estimated with a priori value of 1.4
Ionospheric refraction correction	No	No
Tropospheric refraction correction	Yes	Yes
Antenna mount correction	No	No
Polar motion correction	Yes	Yes
Earth tides	No	No

Summary of Batch Least-Squares Estimation Results

Fig. 2 summarizes the RMS and maximum position differences during the overlap periods. The overlap RMS is nearly constant at 1.5 meters (m) with the exception of the October 25–October 28 solution overlap, in which case the comparison was 2.5 m. This overlap resulted in an average total position RMS of 1.7 ± 0.5 m. During this time period, the maximum position differences per overlap ranged from about 2.5 to 5.5 m, typically close to the expected values, which are twice the RMS value for the corresponding overlap comparison. The exception to this is the overlap of the October 19 and October 22 solutions, which yielded a 5.5-m maximum. In this case, the wide separation between the maximum position difference and the RMS difference is the result of significantly differing solve-for thrust coefficients for that time period in the two solutions. Fig. 3 gives the solved-for thrust coefficients, of the form $1 + \tau$ times the nominal 1 micronewton applied thrust. The average of 4 overlap maximum position differences was 4.1 ± 1.4 m. Generally, the maximum position differences occur at the ends of the definitive data arcs where the greatest difference exists in the solve-for thrust coefficients, while the differences nearer to the center of the data arcs are similar to the RMS position differences, and are more representative of the solution consistency.

As a result of the altimetric goals of the TOPEX/Poseidon mission, the radial accuracy of the precision ephemerides used for the science processing is 13 cm. The maximum and RMS overlap radial differences are given in Fig 4. The RMS values varied from less than 10 cm to just over 40 cm, with the maximum differences ranging from 15 to 90 cm. As with the total position differences, the exception being the overlap of the October 19 and October 22 solutions. The average RMS radial position difference is 0.31 ± 0.13 m, which is approximately three times the requirement for the precision ephemerides.

The comparisons of the overlap velocity comparisons are presented in Fig. 5. The distribution of the maximum and RMS differences is virtually identical to the total position overlaps, as was expected. The average overlap total velocity difference RMS is 1.5 ± 0.5 millimeters (mm)/second (s), and the average maximum was 3.4 ± 1.0 mm/s. Once again, the average RMS value was representative of the consistency over the entire overlap period.

Solution measurement residuals for the one- and two-way Doppler tracking data used are presented in Fig. 6. For all of the solutions the two-way Doppler residual statistics were generally consistent, with the mean of the residuals averaging 1.2 ± 0.4 mHz and the standard deviation of the residuals averaging 29.2 ± 2.5 mHz. The mean residual is much smaller than the standard deviation, indicating that no significant biases exist in the measurements. Because the USO bias and drift were estimated, the mean one-way residuals were expected to be insignificant, and the resulting mean residual was virtually zero. The standard deviation of the one-way residuals was approximately 60 percent that of the two-way, averaging 17.1 ± 1.5 mHz. Because the one-way data travel half the path of the two-way data, the one-way data noise is expected to be greater than 50 percent that of the two-way noise if the two processes are not fully correlated.

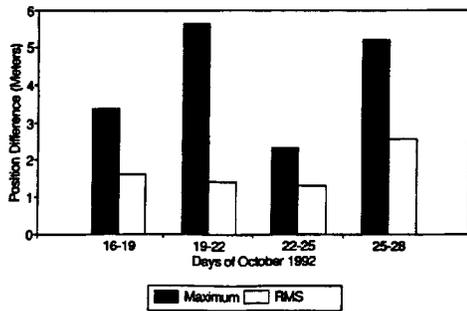


Fig. 2 TOPEX Overlap Comparisons

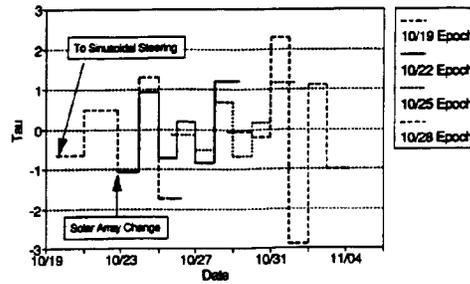


Fig. 3 Estimated Thrust Coefficients

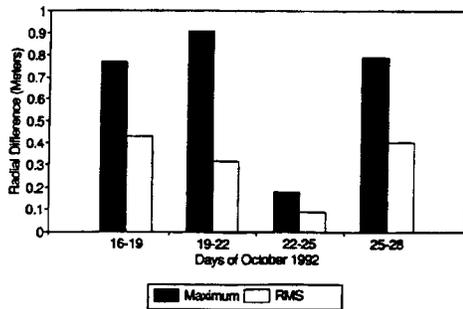


Fig. 4 Radial Overlap Comparisons

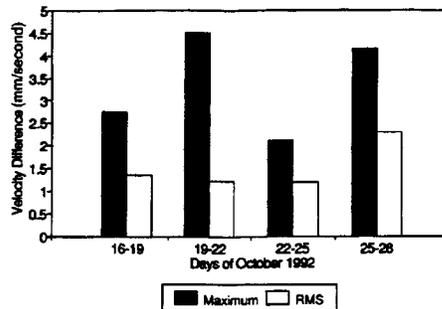


Fig. 5 Velocity Overlap Comparisons

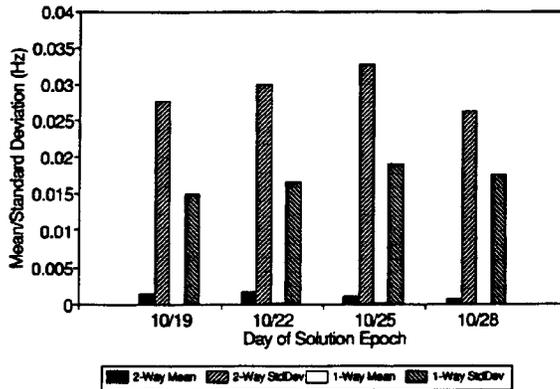


Fig. 6 Doppler Residual Statistics

In summary, the enhanced GTDS solutions show an average consistency of 1.7 m in position and 1.5 mm/s in velocity, over the definitive data arcs. At the ends of the data arcs, maximum variations reached 4.1 m and 3.4 mm/s. The majority of the differences are attributable in part to differences in the solve-for thrust estimation factors near the end of the arcs. The Doppler residuals were low for the solutions, being on the order of 20 mHz with a near-zero mean.

Summary of Sequential Estimation Results

Several indicators were available to assess the quality of the RTOD/E solutions independently of other orbit determination systems. Among such performance criteria are the diagonal components of the state error covariance matrix, more specifically, the square root of these values (standard deviation). Fig. 7 shows the time-evolution of the 1- σ root-sum-square (RSS) position error for each of the three satellites during the Cycle 4 period, as computed by RTOD/E. During Cycle 4, the average 3- σ position error for TOPEX was 1.3 m, 5.4 m, and 3.6 m in radial, along-track, and cross-track components, respectively. The relative constancy shown in the figures indicates that RTOD/E solutions had largely settled from earlier maneuver perturbations and initial condition errors. The two spikes apparent in the TDRS-West profile correspond to a pair of burns used for a plane-change maneuver. This sudden change in the orbital state root variance was a direct consequence of the application of assumed delta-V uncertainty to the covariance matrix. As reflected in the impulsive character of the variations, the recovery time was minimal. Overall, a gradual trend toward reduced levels in standard deviations is seen for each relay satellite.

Additional evidence of solution consistency is provided by the size of the predicted residual for the first tracking measurement in a pass. The values for the first TDRSS one- and two-way Doppler measurements for each tracking pass for a typical day are provided in Fig. 8. The first residual of each pass on October 27 was within the 3 σ bound in the residual space.

Trends in the estimates for the coefficients of solar radiation pressure (TDRS and TOPEX) and the coefficient atmospheric drag (TOPEX) can be seen in Figs. 9 and Fig. 10. Mean values over the 10-day span for the coefficient of atmospheric drag (C_D) for the coefficient of the solar radiation pressure for (C_R) for TOPEX, the coefficient of the solar radiation pressure for (C_R) for TDRS-Spare the coefficient of the solar radiation pressure for (C_R) for TDRS-West estimates are 1.85 ± 0.12 , 1.05 ± 0.08 , and 1.39 ± 0.03 , and 1.44 ± 0.04 respectively. It is reasonable, though not essential, to expect the variations in the estimates for these parameters to correlate to uncertainty in the atmospheric density modeling due to variation in the solar flux level. It should be noted that the uncertainties in the modeling of atmospheric densities are greater for spacecraft at higher altitudes than for spacecraft at lower altitudes. The percent change from mean of daily F10.7 solar flux during the Cycle 4 time period was approximately 50 percent. The variations from mean in TOPEX C_D , and TOPEX C_R , TDRS-Spare C_R , and TDRS-West C_R estimates were 31, 35, 12, and 11 percent, respectively.

Other parameters that provide a basis for performance assessment are Doppler and USO bias estimates. Variations in the measurement bias estimates are largely induced by such unmodeled physical phenomenon as ionospheric and tropospheric refraction and transponder delay. The estimates for the USO clock bias are

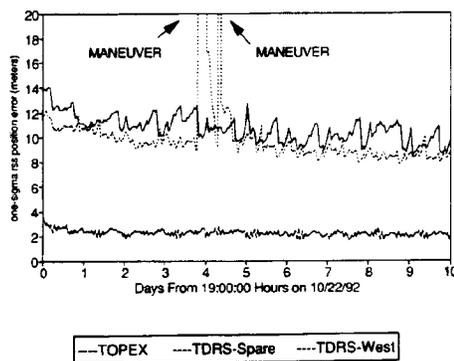


Fig. 7 1- σ Position Uncertainty

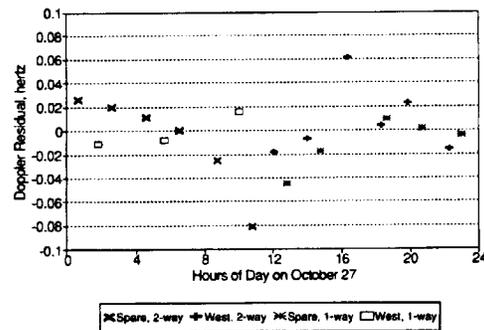


Fig 8 Residual Values for the First Measurements of Each TOPEX Tracking Pass

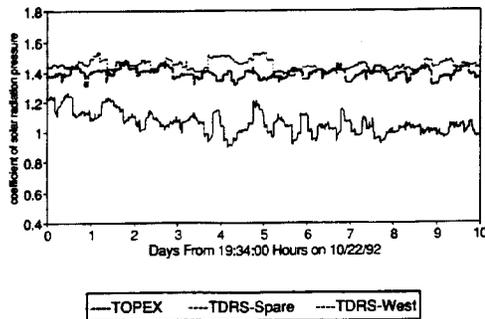


Fig. 9 Coefficients of Solar Radiation Pressure

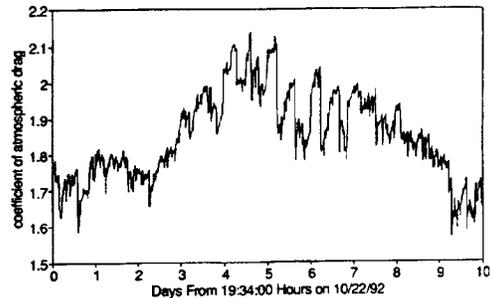


Fig. 10 Coefficient of Atmospheric Drag for TOPEX

consistent with corresponding estimates generated independently by the batch least-squares orbit determination process.

Results of POEs and GTDS Solutions Comparisons

Four GTDS ephemerides were compared to the common timespans within the preliminary Cycle 4 POE. The timespans of the GTDS definitive ephemerides are given in Table 4.

The second and third GTDS ephemerides lie completely within the timespan of the Cycle 4 POE, while the first and fourth GTDS ephemerides overlap the beginning and end of the Cycle 4 POE, respectively. The ephemerides were compared at 10-min intervals in orbit plane coordinates on their common definitive spans. The RSS position differences between the GTDS ephemerides and the Cycle 4 POE are shown in Fig. 11. The average position difference is on the order of 5 to 6 m, with the orbital maximum differences being less than or equal to 9 m.

Fig. 12 shows the differences in the radial, cross-track, and along-track directions on October 27, 1992. The maximum radial differences are less than 1.5 m, while the maximum along-track differences are less than 5 m. The cross-track differences, which are the largest of the three components, are about 9 m. The differences in the along-track component have an average value of 2 m, while the average differences in the radial and cross-track components are nearly zero. The trends in the component-by-component comparisons are similar for all GTDS ephemerides.

Further analysis of the cross-track component showed that the smallest differences occur at the orbital nodes, while the largest differences occur at the maximum latitudes, indicating a disparity in inclination. The modeling differences between GEODYN and GTDS are being analyzed in an effort to identify the source of this systematic orbit plane difference in the cross-track direction.

Comparison Between POEs and Sequential Ephemerides

The ephemeris comparison results are illustrated in Figs. 13 and 14. Fig. 13 shows the root-sum-square position difference between the POEs and RTOD/E ephemerides over the Cycle 4 interval. Fig. 14 shows radial, cross-track, and along-track components of the position difference between POEs and RTOD/E ephemerides during a representative day (October 27). The position difference grows to approximately 19 m near the middle of the 10-day cycle and reduces to approximately 10 m near the beginning and end of the cycle. In addition, a slight 24-hr modulation of approximately 2 m is visible. As Fig. 14 shows, the total difference is dominated by the cross-track component. The maximum differences observed during the 10-day cycle are 18.6 m, 2 m, and 9.1 m for cross-track, radial, and along-track components, respectively. A bias of approximately -2 m was observed in the along-track component of the position difference.

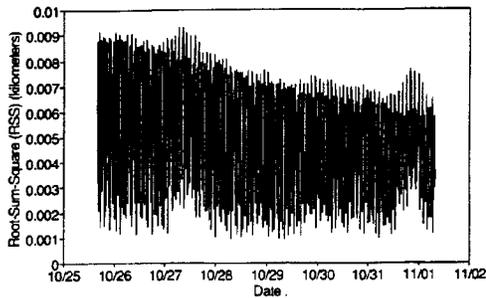


Fig. 11 Position Differences Between the Cycle 4 POEs and GTDS Ephemeris

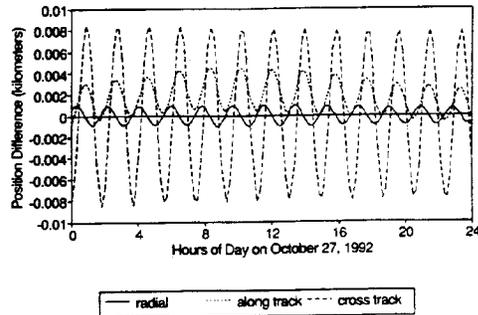


Fig. 12 Radial Position Differences Between the Cycle 4 POEs and GTDS Ephemeris

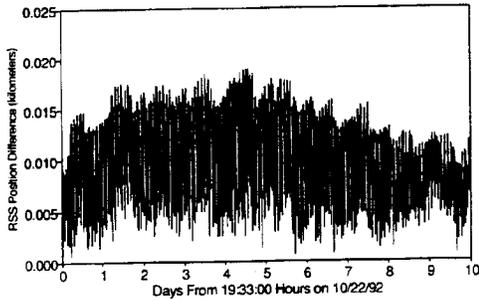


Fig. 13 Position Difference Between the Cycle 4 POEs and RTOD/E Ephemeris

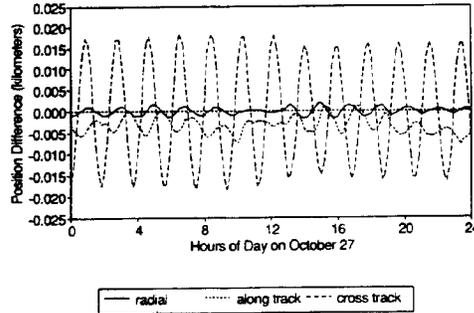


Fig. 14 Position Difference Between the Cycle 4 POEs and RTOD/E Ephemeris

In an attempt to identify the sources of cross-track discrepancy, the comparison was repeated in an Earth-fixed coordinate frame. The RSS position difference envelope was reduced by approximately 5 m and exhibited no 24-hr modulation. This condition suggests that a significant portion of the cross-track difference arose from discrepancies in Earth orientation modeling TOD. Thus, the effects on the ephemeris comparison of Greenwich hour angle (GHA) and polar motion angle discrepancies are being analyzed. As a preliminary step, the GHA values computed by the two orbit determination systems were compared over the relevant time period. A mean GHA difference was then computed (2.6×10^{-7} radians) and applied to the RTOD/E ephemeris as a constant rotation about the z-axis. The maximum difference for the adjusted cross-track comparison was only slightly smaller (by approximately 1 m). The RTOD/E 3- σ position uncertainties in the along-track and radial directions are larger than the differences between RTOD/E ephemerides and POEs; that is not the case in the cross-track component.

Comparison Between GTDS Batch Least-Squares and Sequential Ephemerides

The ephemeris comparison results are illustrated in Figs. 15 and 16. Fig. 15 shows the RSS position difference between a representative GTDS definitive ephemeris (solution epoch on October 25) and the RTOD/E ephemeris over a 6.6-day portion of Cycle 4. Fig. 16 shows radial, cross-track, and along-track components of the position difference over 1 particular day. The RSS position difference grows to a maximum of approximately 14 m. The along-track, cross-track, and radial components reach maximum values of 11.6, 2.9, and 10.4 meters, respectively. The average GHA discrepancy between GTDS and GEODYN was

approximately 1.4×10^{-8} radians, an order of magnitude smaller than the GEODYN-RTOD/E GHA discrepancy.

Remarks on Supporting Analysis

Batch least-squares covariance analysis was performed to analyze the GTDS solutions.⁸ The covariance analysis simulations were performed corresponding to the GTDS solution with an epoch on October 28, 1992. The modeling for covariance analysis was made as close as possible to the GTDS modeling. The 3- σ RSS position uncertainty was found to vary between 7 and 12 m. By components, the maximum 3- σ position uncertainties were 2.5 m, 6 m, and 11 m in radial, cross-track, and along-track directions, respectively. The radial and along-track differences between GTDS solutions and POEs are less than the uncertainties obtained by covariance analysis; this was not the case with the cross-track component. These results are being analyzed further. At the maximum 3- σ RSS position uncertainty of 12.3 m, the major contributors to the errors are the uncertainty in the ionospheric refraction correction at WSGT (9.6 m) and the geopotential (5.4 m).

Several areas in the batch least-squares modeling and orbit determination processing could be improved to yield better results. First, the TDRS orbit determination and the solution orbits used in the observation modeling could be improved. Second, the area modeling of TOPEX itself should be improved. At present only average areas are used for solar radiation and drag force computations. Also, the solar radiation pressure modeling only included the component of the force in the anti-sunward direction, neglecting a potentially substantial normal component. Finally, better treatment of the along-track portion of the unmodeled body-fixed force should help improve the accuracy of the batch least-squares solutions.

The nature of the FDF/POE comparisons indicates that a coordinate system difference could exist between GTDS, RTOD/E, and GEODYNE. The maximum cross-track differences occur near the orbit north/south points, indicating an inclination differences. To better understand and confirm the nature of the differences, an after-the-fact rotation was applied to the GTDS ephemerides to lower the inclination by .214 arc-sec. As anticipated, this virtually eliminated the cross-track differences, and reduced the average RSS position difference to 1.9 m. The remaining differences were dominated by the along-track difference, while some residual cross-track differences remained, possibly from a slight difference in right ascension.

There are two possible sources for the inclination differences. The first is a difference in the coordinate system implementations. The second is a difference in the modeling of the station locations, reference geoid, or the tracking measurements. Comparison of the GTDS and GEODYNE Mean-of-2000 to true-of-date to Earth-fixed rotations, showed that the systems agree in the rotations. This leaves the station location and measurement modeling. Review of the models used indicates that there is agreement in the shape of the Earth. Therefore, the cause of the differences is a real effect of the measurement types and modeling being used in the orbit solutions. This is further supported by the error analysis results discussed above.

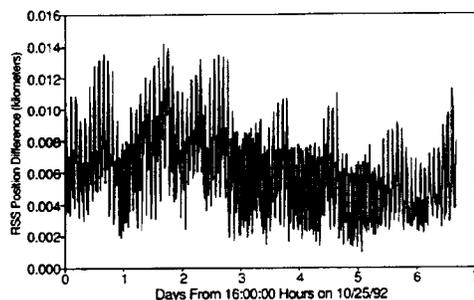


Fig. 15 Position Difference Between GTDS and RTOD/E Ephemeris

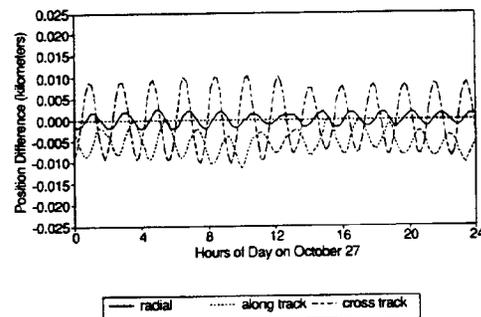


Fig. 16 Position Difference Between GTDS and RTOD/E Ephemeris

It is important to note that if TDRSS-tracking is not expected to yield orbit solutions with accuracy comparable to laser-tracked orbit solutions.

CONCLUSIONS

This study analyzed the TDRSS user orbit determination accuracy using a batch least-squares method and a sequential estimation method. Independent assessments were performed of the orbit determination consistency within each method, and the estimated orbits obtained by the two methods were compared. This assessment is applicable to the timeframe studied here.

In the batch least-squares analysis, the orbit determination consistency for TOPEX/Poseidon, which was heavily tracked by TDRSS, was found to be about 2 m in the RMS overlap comparisons and about 5 m in the maximum position differences in overlap comparisons. In the sequential analysis, the 3- σ state error covariance function for RSS position was found to be under 12 m. As a measure of consistency, the first residual of each pass was within the 3- σ bound in the residual space.

The differences between the definitive batch least-squares ephemerides and the POEs were no larger than 10 m. The differences between the forward-filtered sequentially estimated ephemerides were no larger than 19 m. The dominant component in the differences was in the cross-track direction. Further analysis is in progress to understand the magnitudes of the differences.

The differences among the POEs, GTDS, and RTOD/E solutions can be traced to differences in modeling and tracking data types, which are being analyzed in detail. As more precise POEs become available, further comparisons and analysis will be performed.

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