Orbit Determination Requirements for Topex

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ORBIT DETERMINATION REQUIREMENTS FOR TOPEX

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The ability of radar altimeters to measure the distance from a satellite to the ocean surface with a precision of the order of 2 cm imposes unique requirements for the orbit determination accuracy. The orbit accuracy requirements will be especially demanding for the joint NASA/CNES Ocean Topography Experiment (Topex/Poseidon). For this mission, a radial orbit accuracy of 13 centimeters will be required for a mission period of three to five years. This is an order of magnitude improvement in the accuracy achieved during any previous satellite mission. This investigation considers the factors which limit the orbit accuracy for the Topex mission. Particular error sources which are considered include the geopotential, the solar radiation pressure and the atmospheric drag model, as well as the effects introduced by variations with aspect angle in the spacecraft area-to-mass ratio. Finally, the activities of the teams organized to perform the model and software improvement for meeting the Topex objective are reviewed.

INTRODUCTION

The transport of water in the ocean has an influence on the earth's climate, weather and food supply, and provides a means of dispensing pollutants and other wastes. However, because of the size of the earth's ocean, it has been difficult to observe, and hence, it is poorly understood. Satellites, carrying appropriate sensors, provide the capability to globally monitor the ocean with a temporal sampling of a few days to a month. Both active and passive sensors are used for this purpose and have led to the rapidly developing field of satellite oceanography [*Stewart*, 1985]. The satellite-borne radar altimeter has evolved into one of the fundamental instruments for satellite oceanography applications.

The development of satellite altimeters for the active sensing of ocean surface topography has been one of the primary objectives of the NASA Ocean Processes Program for over a decade. The requirements for NASA's satellite altimetry program were formulated at the 1969 Williamstown Conference on Solid Earth and Ocean Physics [*Kaula et al.*, 1970]. Since this conference, satellite altimeters have flown on Skylab [*McGoogan et al.*, 1974], GEOS-3 [*Stanley*, 1980], Seasat [*Tapley et al.*, 1981], and Geosat [*Mitchell and Hallock*, 1984]. The importance of the altimeter for oceanographic measurements is summarized by *Wunsch and Gaposchkin*, [1980]. Characteristics unique to satellite altimetry are: 1) the high accuracy with which surface topography, wave height and wind speed can be measured and 2) the ability to collect a global set of these measurements in the time interval of a few days.

The oceanographic phenomena which influence the surface topography include currents, mesoscale eddies, tides, storm surges and the marine geoid [*Apel*, 1980]. Given an independent determination of the satellite height from the orbit determination system and an accurate marine geoid, the ocean surface topography can be inferred. Altimeter measurements also can be used to define and monitor changes in the topography of the polar ice caps [*Brooks et al.*, 1978]. The comprehensive volume of *JGR* articles on the GEOS-3 [*Scientific Results of the Geos-3 Mission*, 1979] and Seasat [*JGR Special Issue I*, 1981; *JGR Special Issue II*, 1983] missions describe numerous successful applications of satellite altimetry.

As a follow-on to the Seasat mission, NASA and CNES have agreed to collaborate in a joint mission to use a satellite to measure the ocean surface topography over entire ocean basins for a period of three to five years. The ocean topography will be determined using a satellite altimeter which has the ability to measure the distance from the satellite to the ocean surface with an instrument precision of approximately two centimeters. The measurements obtained during this mission will be integrated with subsurface oceanographic measurements and models of the ocean's density field to determine the general circulation of the ocean and its variability. The rationale for the mission and its role in the general ocean topography experiment, which is referred to as Topex by NASA and Poseidon by CNES, are given in the report of the *Topex Science Working Group* [1981] and in the Topex/Poseidon Science Opportunities document [*Stewart et al.*, 1986]. For satellite altimetry applications, the satellite serves as a stable platform from which the altimeter measures the average distance from the antenna feed point to the instantaneous electromagnetic mean sea level.

Figure 1 illustrates the satellite altimeter measurement. As the satellite flies over the ocean surface, the distance from the satellite to the ocean surface is inferred from the time of flight of radar pulses transmitted from the satellite and reflected from the ocean surface. Note that the ocean surface will contain both static or constant variations, the geoid, and time-varying changes. Given the geoid and the altimeter measurement, the time-varying ocean surface changes can be determined. These changes are due to the effects of tides, currents, waves and other high-frequency ocean surface variability.

A necessary requirement to be able to use the satellite altimeter measurements in modeling the ocean surface is that the satellite orbit be computed with an accuracy comparable to the accuracy of the satellite altimeter measurement. For the Topex/Poseidon mission, the radial component of the satellite's orbit must be known to the order of 14 cm rms. Since the maximum mission lifetime for Topex is five years, this imposes a very strenuous orbit determination requirement. This requirement is especially significant when it is viewed in light of the fact that at the initiation of planning for the Topex mission the radial component of the Seasat orbit, following extensive efforts to compute a precise ephemeris, was known to the order of 70 cm to 1 m in the radial component [*Schutz et al.*, 1981]. The radial orbit error was further improved to the 50 cm level [*Schutz et al.*, 1985]. While this represents approximately an order of magnitude improvement in the radial orbit accuracy from the initial Seasat orbit determination, it is approximately an order of magnitude worse than the requirements for Topex. This presentation describes the mission, discusses the requirements for precise orbit determination, and describes the current efforts which are being conducted to satisfy this requirement.

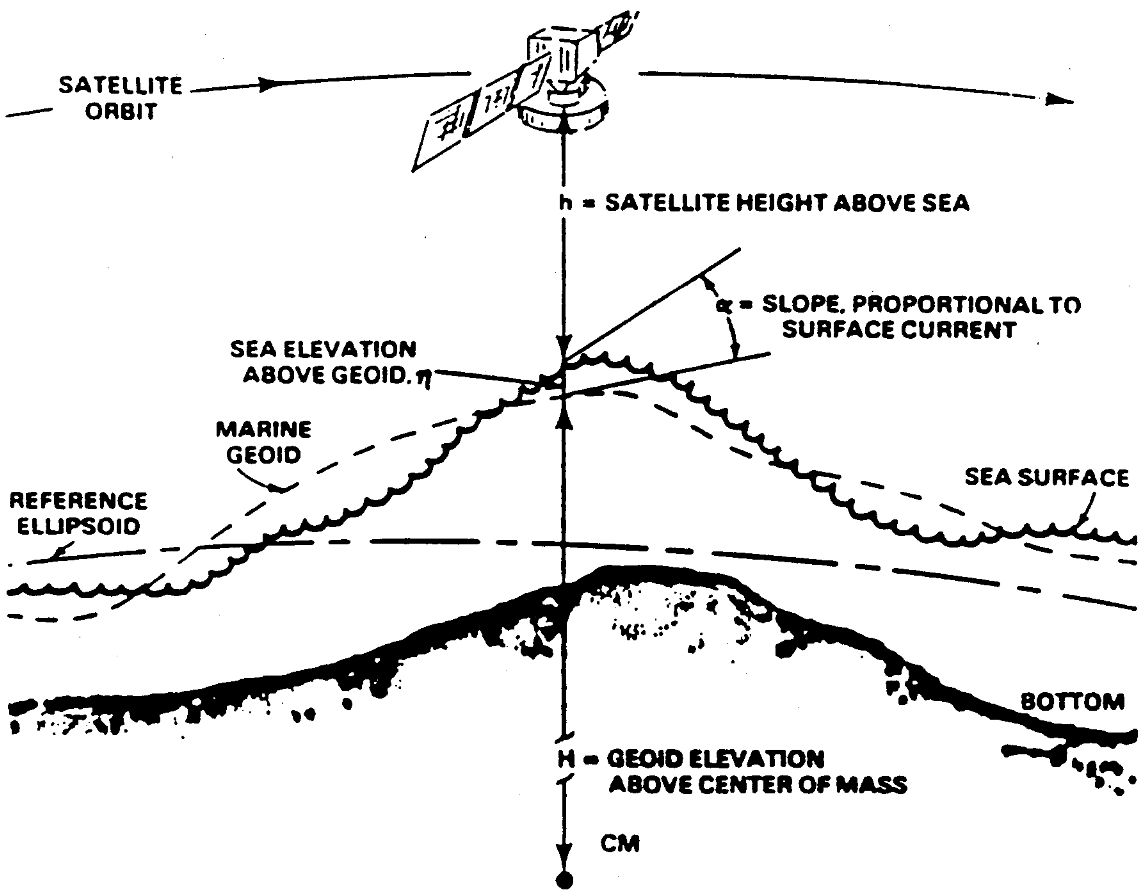

Figure 1. The Satellite Altimeter Measurement

THE TOPEX MISSION

The Topex satellite will orbit with an altitude of 1333 km and an inclination of 63.13°. At this altitude, the effects of atmospheric drag will be minimized, and the influence of errors in the earth's gravity field will be attenuated. However, the effects of the solar radiation forces will be larger than the effects at lower altitudes. The 63.13° inclination is selected to provide large values for the intersection angles between ascending and descending ground tracks at midlatitudes, to provide coverage of as much of the world's oceans as possible, and to minimize the aliasing of the ocean tide components into the measured ocean surface topography. Finally, the orbit characteristics are selected so that the ground track will repeat to within ±1 km once every ten days. The repeated ground track is essential to allow long duration averaging of temporal effects in the ocean surface. The nominal mission orbit elements are summarized in Table 1.

TABLE 1 NOMINAL TOPEX ORBIT ELEMENTS	
Semi-major axis	7712190 meters
Eccentricity	.0015
Inclination	63.13 degrees
Perigee motion	0 degrees/day
Node motion	–2.29 degrees/day
Mean motion	4613.6 degrees/day

To separate variations in the satellite height from variations in the ocean surface height and to orient the ocean surface height measurements with respect to a common center-of-mass coordinate system, an independent determination of the radial component of the satellite's orbit is determined from ground-based tracking of the spacecraft. In addition to the two high precision radar altimeters, the instruments carried by Topex to support the orbit determination include a Tranet beacon to provide the primary tracking of the satellite, a laser retroreflector assembly to allow tracking from ground-based lasers to verify the altimeter measurements of height and to provide supplemental precision tracking of the satellite, and a Global Positioning System (GPS) satellite receiver to provide data for testing new methods for accurate tracking of the satellite's position. In addition, CNES will provide a radiometric tracking system using a one-way, two-frequency doppler system called Doris that receives signals transmitted by beacons on the ground. All of the radio instruments except the CNES altimeter will be two-frequency instruments to allow correction for the ionospheric refraction. Finally, to obtain the correction for the wet troposphere, a two-frequency radiometer will be carried onboard. The characteristics of these sensors are summarized in Table 2.

The orbit determination accuracy will depend on the knowledge of the dynamic forces which act on the satellite, the accuracy of the tracking station coordinates and the accuracy and frequency with which the satellite's motion can be observed. The dominant forces which influence the satellite's orbit are due to the gravitational attraction of the earth, sun and moon, the effects of atmospheric resistance and the influence of the direct and reflected solar radiation pressure. Each of these effects must be modeled with sufficient accuracy to meet the orbit computation requirements. Other effects which influence the orbit accuracy include geographic and temporal distribution of the tracking data and the sophistication of the software system used to determine the estimates of the orbit. A detailed summary of the effort which must be expended to compute precise satellite orbits for geodetic and oceanographic satellites is

TABLE 2. TOPEX/POSEIDON INSTRUMENTS				
System	Instrument	Purpose	Frequency	Bandwidth
Topex	Altimeter	Measures height of satellite above the sea, wind speed, wave height, and ionospheric correction	5.3 GHz 13.6 GHz	320 MHz 320 MHz
	Radiometer	Measures water vapor along the path viewed by the altimeter, which is used to correct the altimeter for pulse delay due to water vapor	18.0 GHz 21.0 GHz 37.0 GHz	220 MHz 220 MHz 220 MHz
	Tranet beacon	Provides doppler signal for Tranet ground stations for precision orbit determination	400.0 MHz 150.0 MHz	—
	GPS receiver*	Provides a new tracking data type (range differences) for precision orbit determination	1227.6 MHz 1575.4 MHz	—
	Laser retroreflector	Used with ground-based lasers to calibrate and verify altimeter measurements of height	—	—
Poseidon	Altimeter*	Measures height of satellite above the sea, wind speed, and wave height	13.65 GHz	330 MHz
	Doris	Receives signals from ground stations for satellite tracking, gravity field measurements, and ionospheric correction for altimeter	401.25 MHz 2036.25 MHz	150 KHz 50 KHz

* Experimental instruments

summarized in the *Journal of Astronautical Sciences* [Seasat Ephemeris Analysis, Special Issue, 1980] and [*Tapley et al.*, 1979].

TOPEX ORBIT DETERMINATION

The 14 cm rms radial height accuracy which the Topex mission is required to satisfy presents a unique and extremely difficult challenge to the field of precision orbit determination. Conceptually, the problem can be formulated as a nonlinear parameter estimation problem which must be solved numerically through an iterative procedure similar to the one outlined in Appendix A. A careful consideration of both the orbit computation methodology and the tracking system is required to achieve this goal. The overall Topex height error budget allocation is summarized in Table 3. The error budget is based on an intensive analysis of the dynamic and measurement model errors and is the residual error after certain *a priori* dynamic model parameters are adjusted using postlaunch tracking. Note that the predominant portion of the height error budget is assigned to the orbit error. The baseline tracking system for the Topex mission was based on a 40-station network of improved Tranet-II doppler tracking stations. The improvement implies that the oscillators for both the satellite beacon and the ground-based receivers will be stable at the level of 5×10^{-13} for periods of up to 1000 secs. This stability will allow the range-rate data to be processed as integrated range difference data with an inherent measurement precision of the order of 5 cm. Data of this type from the Tranet tracking network have not been used for general orbit determination purposes.

The single largest error source in the error budget is due to the error in the earth's gravity field model. In addition to contributing to the long period and secular growth in the radial orbit error, the short period error (on the order of one revolution or less) has a geographically correlated component that is particularly insidious for satellite altimetry applications. [*Tapley and Rosborough*, 1985; *Rosborough*, 1986]. These investigations demonstrate that the gravity field induced short period orbit error will have a component which is geographically correlated and, furthermore, it will vary in phase with the long wavelength components of the geoid error. Accurate satellite tracking using ground-based tracking systems can be used to remove the long period and resonant orbit error components. These components can be removed by breaking the ephemeris up into a sequence of independent arcs of a few days (normally two to ten days) and adjusting the satellite position and velocity at each arc epoch. The daily and short period effects cannot be eliminated in this manner, unless nearly continuous tracking of the satellite is available to allow solutions of the satellite ephemeris for arc lengths less than one orbit revolution.

Since the geopotential model error is expected to be the dominant contributor to the satellite orbit error, particular emphasis was devoted to this error source in the pre-mission planning. A special Topex gravity model improvement effort was initiated in 1985 to achieve the value of 10 cm rms for the gravity error-induced radial orbit error contribution shown in Table 3. Without this effort, the error will be much larger. Effects which are closely related to the gravity model include errors in ocean tides as well as errors in tracking station coordinates and the fundamental terrestrial reference frame. The definition of these effects will be an integral part of the gravity model determination.

The effects of errors in the models for atmospheric drag and solar radiation pressure are factors which must be considered. The specific question regarding the nongravitational forces is whether they can be modeled *a priori* with sufficient fidelity to meet the orbit accuracy requirements or whether special in-orbit force compensation or acceleration measurement techniques will be required.

TABLE 3. ERROR BUDGET FOR TOPEX/POSEIDON MEASUREMENTS OF SEA LEVEL

Error Source	Standard Deviation of Uncertainty, cm	Decorrelation Distance, km
Altimeter		
Instrument noise	2.0	20
Bias drift	2.0	(many days)
Media		
EM bias	2.0	20-1000
Skewness	1.0	20-1000
Troposphere, dry	0.7	1,000
Troposphere, wet	1.2	50
Ionosphere	1.3 (2.0*)	20
Orbit		
Atmospheric drag	1.0	>10,000
Solar radiation	1.0	10,000
Earth radiation	<1.0	10,000
GM	2.0	10,000
Gravity	10.0	10,000
Earth and ocean tides	1.0	10,000
Station and satellite clock	1.0	10,000
Troposphere	1.0	10,000
Station location	5.0	10,000
Higher order ionosphere	5.0 (1.0†)	10,000
RSS Absolute Error	13.3	

1. Dual-frequency altimeter
2. Dual-frequency radiometer
3. Upgraded Tranet tracking system, 40 stations
4. Altimeter data averaged over 3 s
5. $H_{1/3} = 2$ m, wave skewness $= 0.1$
6. Tabular corrections based on limited waveform-tracker comparisons
7. 1300 km altitude
8. No anomalous data, no rain
9. Improved gravity model (by a factor of two over existing models)
10. ±3 mbar surface pressure from weather charts
11. 100 μs spacecraft clock

* For the one-frequency Poseidon altimeter; inferred from models using data from Doris.

† From Doris tracking data

GEOPOTENTIAL MODEL DEVELOPMENT FOR TOPEX

A critical factor in achieving the error budget shown in Table 3 is the assumption that a significant effort be directed at improving the prelaunch gravity model for the earth. Under the initial assumptions for the Topex mission, this gravity model improvement would come from the NASA Geopotential Research Mission (GRM). With the delay in the plans to fly the GRM, the Topex Project initiated an effort to improve the earth's gravity field model to meet the requirements of the Topex mission. In August 1983, a collaborative effort between the NASA Goddard Space Flight Center (GSFC) Geodynamics Branch and the University of Texas Center for Space Research (UT/CSR) was initiated for a sustained and concentrated effort at improving the geopotential model for Topex. The fundamental assumption on which the gravity model improvement effort is based is that by reprocessing the historical satellite tracking data, supplemented by satellite altimetry data from recent missions, a significant improvement in the quality of the gravity field model can be obtained. The effort will include processing selected sets of tracking data from more than 20 different satellites. The data included in this solution will be optical camera data, satellite laser ranging data, unified S-band data, doppler tracking data, and altimeter height measurements from Seasat. A fundamental requirement for the effort is that the gravity model solution should serve equally well in either the software system at GSFC or the software system at UT/CSR. As a consequence, a substantial effort was required to standardize and evaluate the accuracy of the GSFC software system, GEODYN [*Putney*, 1977], and the UT/CSR software system, UTOPIA [*Schutz and Tapley*, 1980]. An intense software intercomparison effort is underway at the present time. The model effects which are currently being compared to ensure compatibility between the two software systems are listed in Table 4.

TABLE 4. SOFTWARE INTERCOMPARISON CONCERNS

- MODEL EFFECTS
    - Inertial reference frame
        - J2000
        - Nutation corrections
    - Terrestrial reference frames
    - Station coordinate model
        - Epoch values
        - Plate motion model
    - Earth rotation solution
    - Tide model
    - Drag model
        - Atmospheric density model
        - Satellite area model ($C_D A/m$)
    - Solar radiation pressure model

The objective of the software intercomparison is to achieve millimeter-level agreement for software which will be used in computing the orbit of the Topex satellite. With this level of agreement, the final Topex gravity model can be used with comparable accuracy in either system. In addition to the determination of the coefficients for the Topex geopotential model, an extensive effort will be undertaken to evaluate the accuracy of the gravity field model and to quantify the geographically correlated orbit error and assess its impact on the ocean surface topography constructed from the Topex altimeter data. Both analytic and numerical studies will be conducted to achieve these evaluations. The types of intercomparisons planned to validate the gravity model effort are summarized in Table 5.

TABLE 5. GRAVITY MODEL ACCURACY EVALUATION
• Satellite data
• Orbit fits to data used in field
• Orbit fits to withheld data
• Coefficient differences for candidate fields
• Covariance tests
• Geoid evaluations
• Surface gravity
• Altimetry
• Topex data

A particularly important intercomparison will be achieved by comparing surface gravity anomalies derived from the satellite-only gravity field with anomalies determined from surface gravity measurements. The Colorado Center for Astrodynamics Research (CCAR) and the Department of Geodetic Sciences at The Ohio State University (OSU) are collaborating in the studies related to model assessment and validation. To date, the effort has provided gravity models with substantial improvements over previously existing gravity models. The previously published GEM-T1 (GSFC) [*Marsh et al.*, 1986] and the PTGF2 (UT) [*Tapley et al.*, 1987] gravity models indicate, not only the potential for significant improvement of the gravity field using the historic satellite tracking data, but the dramatic potential contained in the satellite altimeter measurements. In this regard, some oceanographers have questioned the use of satellite altimeter data in the gravity model solutions, since it has the potential for aliasing part of the permanent ocean surface topography into the gravity field solution. The use of altimeter crossover measurements, in principal, should not suffer from this effect; however, in order to ensure that there will be no question in the final accuracy of the Topex gravity model, solutions both with altimeter data and without altimeter data will be developed for intercomparison purposes.

THE SURFACE FORCE EFFECTS

After the gravity effects, the next most important dynamic model effects are due to the influence of solar radiation pressure and atmospheric drag. Both of these effects depend on the area-to-mass ratio (A/m) for the satellite. In the original satellite specifications, a small A/m with little area variation was prescribed. Figure 2 shows the Topex satellite as currently configured on the Fairchild bus. Note that there will be significant area variation as the satellite turns to keep the solar panel oriented toward the sun. This effect will complicate both the solar radiation

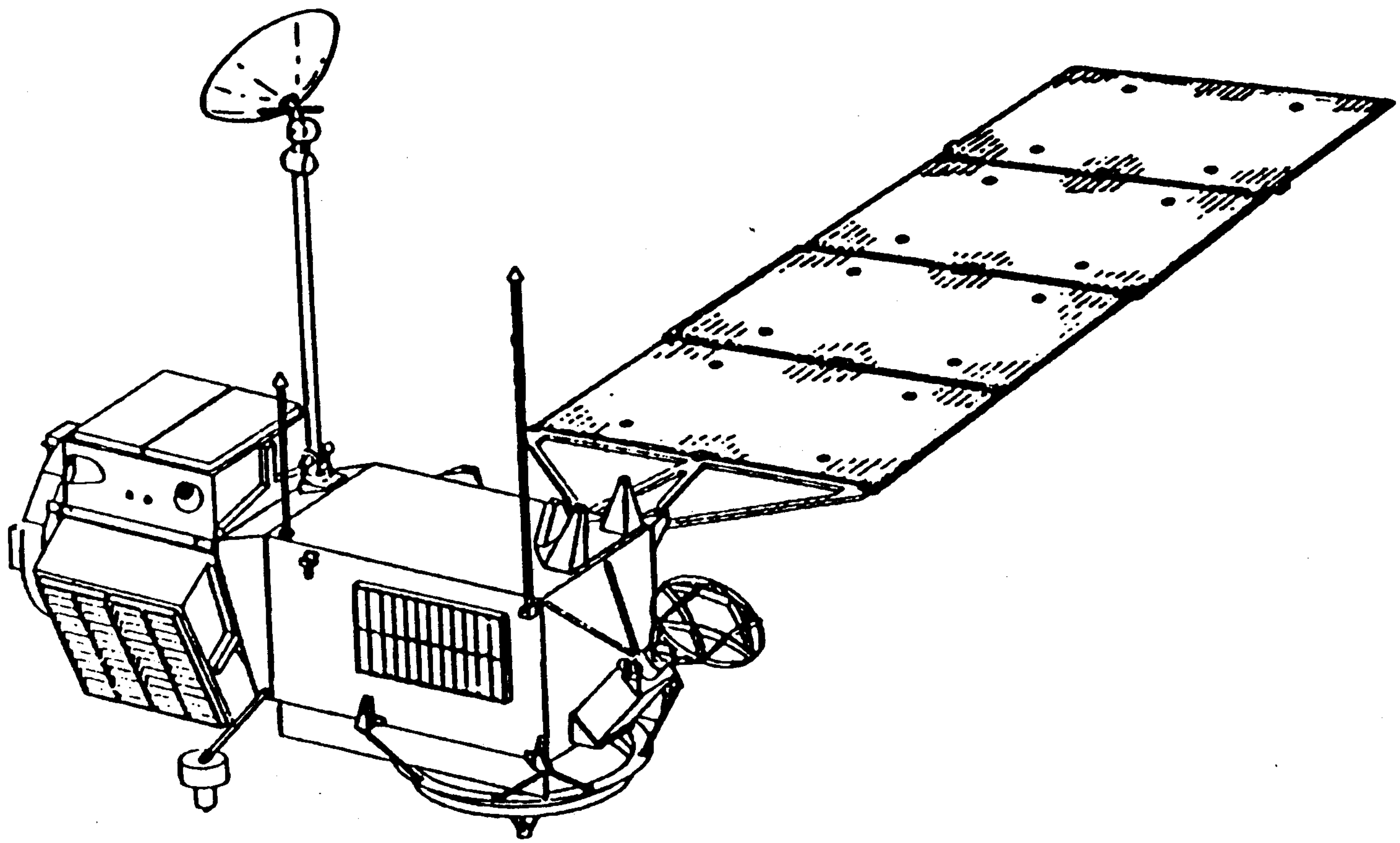

Figure 2. The Topex Satellite

pressure and atmospheric drag modeling.

Solar radiation pressure

The solar radiation pressure is a small force produced when momentum from particles streaming out from the sun is transferred to the satellite. The primary sources of error in the solar radiation pressure model lies in the knowledge of the reflectivity and orientation of exposed surface with respect to the incident radiation. The force due to the direct radiation pressure will be directed away from the sun along the satellite-sun line. The effect of the earth's shadow is a significant factor which must be included. This force for a spherical symmetric satellite is approximately

$$\bar{F} = -P\,(1+\eta)\,\frac{A}{m}\,\nu\,\bar{u} \tag{1}$$

where $\bar{F}$ is the direct solar radiation force per unit mass; P is the momentum flux due to the sun; A is the cross-sectional area of the satellite normal to the sun; $\bar{u}$ is the unit vector pointing from the satellite to the sun; η is the reflectivity coefficient with values between 0 and 1; and ν is an eclipse factor such that $\nu = 0$ if the satellite is in full shadow, $\nu = 1$ if the satellite is in sunlight, $0 < \nu < 1$ if the satellite is in partial shadow. The passage of the satellite from sunlight to shadow is not abrupt, but the interval of time spent in partial shadow will be very brief for near-earth satellites.

Atmospheric Drag

The dominant feature of atmospheric resistance for most satellites is a drag force in the direction opposite to the relative wind. Drag is usually modeled as

$$F\nu_D = -\frac{1}{2}\,\rho\left[\frac{C_D A}{m}\right] V_r^2\,\bar{u} \tag{2}$$

where ρ is the atmospheric density, C_D is the drag coefficient, A is the cross-sectional area perpendicular to $\bar{V}_r$, m is the satellite mass, V_r is the speed relative to the atmosphere, and $\bar{u}$ is a unit vector in the $\bar{V}_r$ direction. The drag force is considerably larger than lift forces perpendicular to $\bar{V}_r$ which may exist. The parameter $(C_D A/m)$ is sometimes referred to as the ballistic coefficient, B, and A/m is the area-to-mass ratio. The drag coefficient, C_D, is a function of the geometry of the satellite and the Mach number, the ratio of the vehicle speed to the speed of sound. In addition, the density ρ is a complicated function, probably best represented by empirical tables.

From the data collected on motions of satellites, especially balloon satellites, and in situ measurements, some basic characteristics of the upper atmosphere density have emerged. These characteristics are:

1. As it rotates, the density undergoes a nearly diurnal variation produced by the sun. The sub-solar region can be modeled with an atmospheric bulge, the axis of which lags slightly behind the earth-sun line.
2. Solar activity has an important influence through production of disturbances in the upper atmosphere. The primary sources of these disturbances are solar flares and solar plasma events. The frequencies of these phenomena follow the eleven-year solar cycle and the 27-day solar rotation period.

3. The density is influenced by geomagnetic activity and interaction with the charged particles in the upper atmosphere. This phenomena can produce significant changes in the density at time scales of a few hours to a day.

4. Seasonal changes in the density, including both annual and semiannual variations, have been observed, although individual models may not account for all these effects.

Most of these phenomena cannot be predicted accurately in advance of their occurrence. The consequence of this difficulty is that considerable uncertainty exists with long-term prediction of satellite lifetimes and that precise orbit computations can only be performed after the fact, i.e., after data have been collected on solar and developed geomagnetic activity.

During the current investigation, three contemporary density models have been evaluated. They are the Jacchia 1971 [*Jacchia*, 1971], Jacchia 1977 [*Jacchia*, 1977] and the CNES DTM [*Barlier et al.*, 1977] models. The primary data used to develop these models were obtained from satellites orbiting at an altitude of 1200 km or lower. As a consequence, the density models applied at the Topex altitude represent extrapolations upward, and as seen in Figure 3, they show significant differences at the Topex altitude.

However, since the mean density decays exponentially with altitude, it is anticipated that the effects of atmospheric density will not be a limiting factor on the Topex orbit error.

CONCLUSIONS

The Topex radial orbit accuracy requirement of 14 cm rms will place unique and previously unrealized constraints on the precision orbit determination accuracy. To achieve this goal, significant improvements in the satellite force model, especially the model for the earth's gravity field, is required. Careful assessment of the primary tracking systems will be necessary to understand and reduce the nature of the systematic errors and to determine accurate locations of the tracking stations. A standardized set of physical constants, reference frame and dynamic model constants as well as an extensive software calibration/validation activity will be required to assure the accuracy of the Topex ephemeris. Significant effort is underway in each of these areas.

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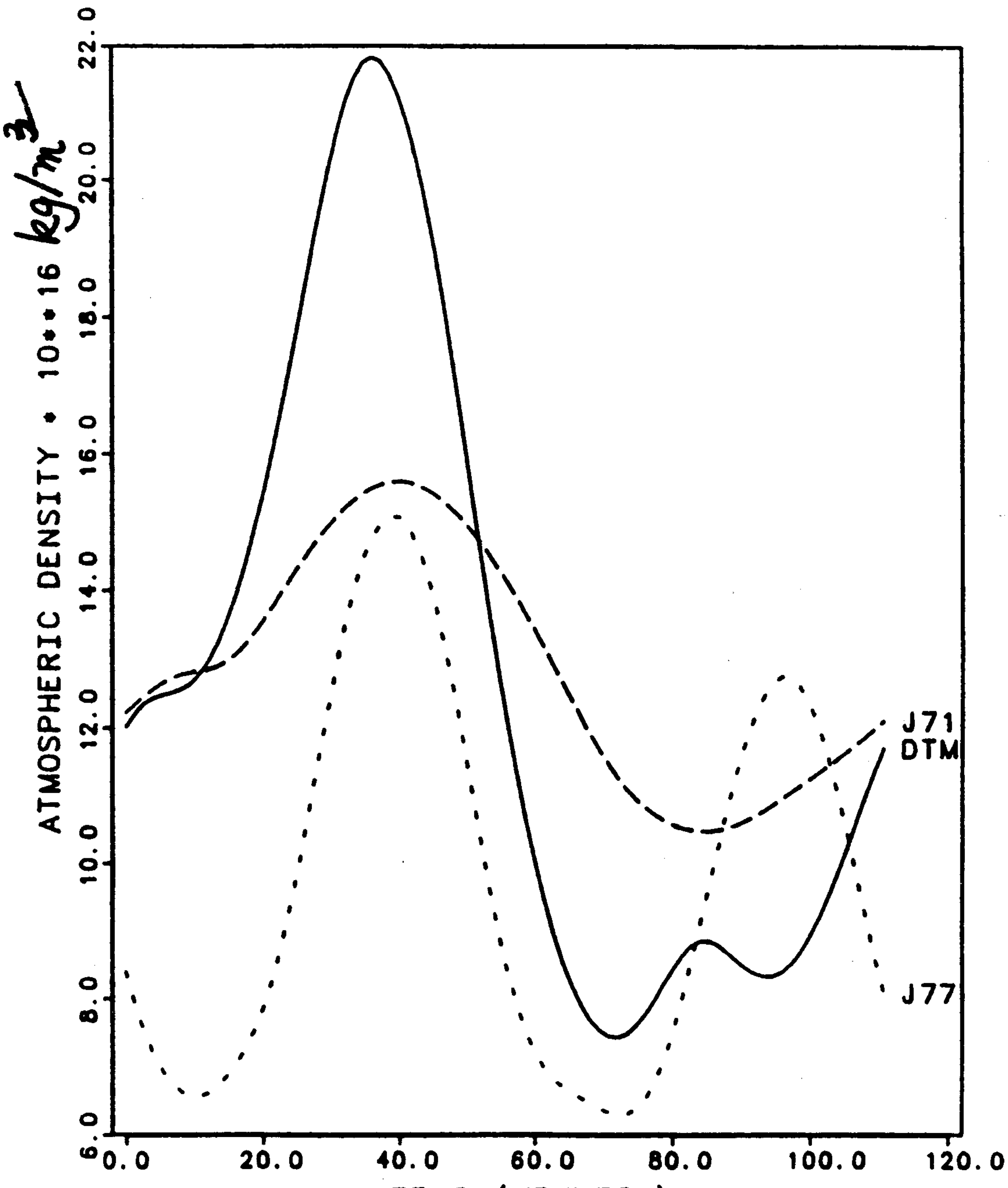

Figure 3. Atmospheric density along one revolution of Topex orbit for various density models (geomagnetic index $K_p = 6$, 10.7 cm solar flux = 115, smoothed solar flux = 131)

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APPENDIX A

THE ORBIT DETERMINATION PROBLEM

In the following, a general statement of the orbit determination problem as it is encountered in most space missions is given. From Newton's law, the vector differential equations of motion can be expressed as

$$\ddot{\bar{r}} = -\mu \frac{\bar{r}}{r^3} + \bar{D}(\dot{\bar{r}}, \bar{r}, t) + \frac{\bar{T}(t)}{m} + \bar{R}(t) \tag{A.1}$$

where $D(\dot{\bar{r}}, \bar{r}, t)$ is the effect of atmospheric drag, $\bar{T}(t)$ is the vehicle thrust and $\bar{R}(t)$ represents the acceleration due to all other forces. Eqs. (A.1) can be expressed in first order form as follows:

$$\dot{\bar{r}} = \bar{v}$$

$$\dot{\bar{v}} = -\mu \frac{\bar{r}}{r^3} + \bar{D}(\bar{v}, \bar{r}, t) + \frac{\bar{T}(t)}{m} + \bar{R}(t) \tag{A.2}$$

$$\dot{m} = -\beta(t)$$

If the vectors $\bar{\xi}$ and $f(\bar{\xi}, t)$ are defined as follows:

$$\bar{\xi} = \begin{bmatrix} x \\ y \\ z \\ u \\ v \\ w \\ m \end{bmatrix} \qquad f(\bar{\xi}, t) = \begin{bmatrix} u \\ v \\ w \\ -\mu x/r^3 + D_x + T_x/m + R_x \\ -\mu y/r^3 + D_y + T_y/m + R_y \\ -\mu z/r^3 + D_z + T_z/m + R_z \\ -\beta(t) \end{bmatrix}$$

then, Eqs. (A.2) can be written as

$$\dot{\bar{\xi}} = f(\bar{\xi}, t) \qquad \bar{\xi}(t_o) = \bar{\xi}_o \tag{A.3}$$

where x, y, z and u, v, w are the components of position and velocity, respectively, with respect to an inertial coordinate system.

If the forces are known exactly (i.e., all components of the force have been modeled correctly and the correct values for all parameters in the force and measurement models are known) and if the orbit injection conditions are known exactly, then Eqs. (A.3) can be integrated to determine the state history $\bar{\xi}(t)$. Generally, $\bar{\xi}_o \neq \bar{\xi}_o^*$, i.e., the true injection conditions do not coincide with the design (or reference) trajectory. Furthermore, even if $\bar{\xi}_o = \bar{\xi}_o^*$, $\bar{\xi}(t) \neq \bar{\xi}^*(t)$, since the mathematical model is not exact.

Therefore, observations or measurements of the vehicle's position must be made after the mission is underway to determine the true trajectory. Since the observations will be influenced by random measurement errors, inaccurate station locations, etc., the determination of the trajectory based on these observations will generally differ from both the true trajectory and the reference trajectory.

Let an augmented state vector $X(t)$ and a force vector $F(t)$ be defined as

$$X = \begin{bmatrix} \bar{\xi} \\ \cdots \\ \alpha \end{bmatrix} \qquad F(X,t) = \begin{bmatrix} f(\bar{\xi},\alpha,t) \\ \cdots \\ 0 \end{bmatrix} \tag{A.4}$$

where α is a q-vector of unknown model constants which satisfy the relation $\dot{\alpha} = 0$. Then Eqs. (A.3) can be rewritten to obtain the differential equations of state as follows:

$$\dot{X} = F(X,t) \qquad X(t_o) = X_o \tag{A.5}$$

Eqs. (A.5) represent a system of n nonlinear first order ordinary differential equations.

Assume that observations have been made at times $t_1, \ldots, t_l$ and that for each t_i, a $p \times 1$ vector of observations, Y_i, has been obtained, where

$$Y_i = G(X_i,t_i) + \varepsilon_i \qquad i = 1, \ldots, l. \tag{A.6}$$

That is, the actual observation, Y_i, is assumed to be a nonlinear function of the true observation, $G(X_i,t_i)$, and the random measurement noise, ε_i.

Now by noting that the solution to Eq. (A.5) can be expressed as

$$X(t_i) = \Theta(X_o,t_o,t_i) \tag{A.7}$$

it follows that

$$Y_i = G(\Theta_i(X_o,t_o,t_i),t_i) + \varepsilon_i$$

or

$$Y_i = \tilde{G}_i(X_o,t_o,t_i) + \varepsilon_i$$

Note that $\tilde{G}_i$ is an implicit relationship. For a general system of differential equations, an explicit relationship usually cannot be determined, since the solution (A.7) cannot be determined in analytic form. Rather, the solution is usually obtained by numerically integrating Eqs. (A.5). Let the m-vectors Y, $\tilde{G}$ and ε, where $m = l \times p$, be defined as

$$Y = \begin{bmatrix} Y_1 \\ \vdots \\ Y_l \end{bmatrix} \quad , \quad \tilde{G} = \begin{bmatrix} \tilde{G}_1(X_o,t_o,t_1) \\ \vdots \\ \tilde{G}_l(X_o,t_o,t_l) \end{bmatrix} \quad , \quad \varepsilon = \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_l \end{bmatrix} \tag{A.8}$$

and the data set or collection of observations can be defined as

$$Y = \tilde{G}(X_o, t_o) + \varepsilon \tag{A.9}$$

In terms of the quantities defined in Eqs. (A.5) and (A.8), the following definition of the state estimation process can be made:

Definition: State Estimation is the problem of determining the "best" estimate, $\hat{X}$, in some sense to be defined, of the state of a vehicle whose initial state, X_o, is inaccurately known using observations, Y, influenced by random observation errors, ε, for a dynamic process whose differential equations are not precisely known.

Examination of Eqs. (A.9) indicates that the relation represents a system of m algebraic equations in terms of the n unknown components of the state and the m unknown components of the observation error. If $\varepsilon_i = 0$, $i = 1,2,\ldots,l$, then any n of Eqs. (A.9) can be used to determine X_o or the state at any other time X_k, through Eqs. (A.7).

If $\varepsilon_i \neq 0$, then some best estimate must be obtained where "best" is used to select one estimate or solution from the many possible solutions. One criterion which has wide acceptance in practice is to minimize the sum of the square of the observation errors. For generality, let the observation state relation be expressed in terms of the state at an arbitrary epoch t_k (e.g., the reference epoch used in Eq. (A.9) is t_o). Then,

$$Y_i = \tilde{G}_i(X_k, t_k, t_i) + \varepsilon_i \qquad i = 1,\ldots,l$$

and let $J(X_k^*)$ be defined as

$$J(X_k^*) = \sum_{i=1}^{l} \varepsilon_i^{*T} \varepsilon_i^* = \sum_{i=1}^{l} [Y_i - \tilde{G}_i(X_k^*, t_k, t_i)]^T [Y_i - \tilde{G}_i(X_k^*, t_k, t_i)] \tag{A.10}$$

Now, let $\hat{X}_k$ be the value of X_k^* which minimizes $J(X_k^*)$. Then, it is necessary that

$$\left.\frac{\partial J}{\partial X_k^*}\right|_{\hat{X}_k} = 0 \qquad \delta X_k^T \left[\frac{\partial^2 J}{\partial X_k^* \partial X_k^*}\right]_{\hat{X}_k} \delta X_k \geq 0\,. \tag{A.11}$$

for arbitrary ∂X_k. From the first of Eqs. (20),

$$\left.\frac{\partial J}{\partial X_k^*}\right|_{\hat{X}_k} = -\sum_{i=1}^{l} [Y_i - \tilde{G}_i(\hat{X}_k, t_k, t_i)]^T \frac{\partial \tilde{G}_i}{\partial \hat{X}_k}(\hat{X}_k, t_k, t_i) = 0 \tag{A.12}$$

Eq. (A.12) is a system of n nonlinear algebraic equations involving the unknown n-vector, $\hat{X}_k$. Eqs. (A.12) must be solved iteratively by a numerical procedure for solving nonlinear algebraic equations. Once $\hat{X}_k$ is known values of $\hat{X}(t)$ can be predicted at any other time, t, by numerically integrating (A.5).

As an alternate approach, Eqs. (A.5) and (A.9) can be linearized and corrections to the reference state, X_k^* can be obtained by iteratively applying the techniques of linear estimation theory [*Tapley*, 1973].