THE INCLUSION OF HIGHER DEGREE AND ORDER GRAVITY TERMS IN THE DESIGN OF A REPEAT GROUND TRACK ORBIT

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Abstract

Mean Orbit Elements can be chosen such that a desired ground track repeat criteria is obtained when propagating with the secular effects of a central body gravity field. If the proper initial conditions are chosen that correspond to these Mean Elements, the orbit can also be propagated with the full gravity field and form another ground track repeat pattern. Further, if the effects of other perturbative forces are compensated for, such that the ground track remains near this pattern, the central body effects can again be considered repetitive. The process of arriving at these conclusions, along with the supporting numerical simulations, are presented.

Introduction

The preliminary design of a satellite orbit that has a repeat ground track is usually done using Mean Elements and some sort of long-term propagator which uses averaged (over one period) equations of motion. When higher accuracy is required such as for the TOPEX/POSEIDON mission, higher order perturbations need to be included. An important distinction can be made in these perturbations. One type has a fixed value for a specific location of the satellite in body-fixed coordinates and thus, because the ground track repeats, also has a repetitive nature. The central body gravity field is the prime example of this type of perturbation. The other type has a temporal variation which is either deterministic (for example, luni-solar perturbations) or of a complex, perhaps random nature (atmospheric drag). The short period effects of these latter perturbations will cause unavoidable deviations from the targeted ground track. The long-term effects can be compensated for by periodic maneuvers to keep the ground track near the target pattern. This paper will be concerned with the first type of perturbation, specifically the effect of higher degree and order gravity terms. By including these effects, the resulting exact repeating ground track will more closely represent the true ground track of the satellite.

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Terminology

To preclude any confusion regarding the terms used when discussing various elements, the following nomenclature will be used. Figure 1 is a cartoon representation of most of the elements discussed below. Pictured is the general relationship over time of a typical orbital element. The scale only gives an indication of the relative sizes of the perturbations. For example, if the orbital element was the semi-major axis of a satellite in the Earth's gravity field, the short period effects of J_2 and the other gravity field terms are about 7 Km and 200 m respectively and only 2 m for the long period variation.

Osculating Elements: These elements are the Keplerian values which represent the true state of the satellite at the specific epoch of interest.

Osculating Minus J_2 (OJ): Some of the major short period effects that contribute to the time varying nature of the Osculating Elements can be removed by various analytical methods. In the case of the Earth, the effect of the oblatenesss (J_2) in the gravity field is three orders of magnitude greater than the effect of any other term. It also has a much simpler analytical representation which can be subtracted from the Osculating Elements to produce the OJ elements. Since the method of Brouwer (Ref. 1) has errors in it, the method of Kozai (Ref. 2) will be inferred. Therefore these elements are equivalent to the ones commonly referred to as "Kozai Mean Elements". Also note that these analytical methods use expressions which are functions of Mean Elements and thus a method of successive approximations has to be used which starts with the osculating values and converges upon the OJ elements.

Mean Elements: These elements represent the elements for which the periodic effects from all perturbations are not present. Thus they can be thought of as "unperturbed elements". However, the remaining secular effects in some of the elements cause these values to be non-constant and thus epoch dependent.

Mean Plus Long Period (M+LP): There are medium and long period effects from various sources. The M+LP are defined as the Mean values plus these effects or conversely the Osculating values minus the short-period terms. Normally, the medium and long period effects due to the

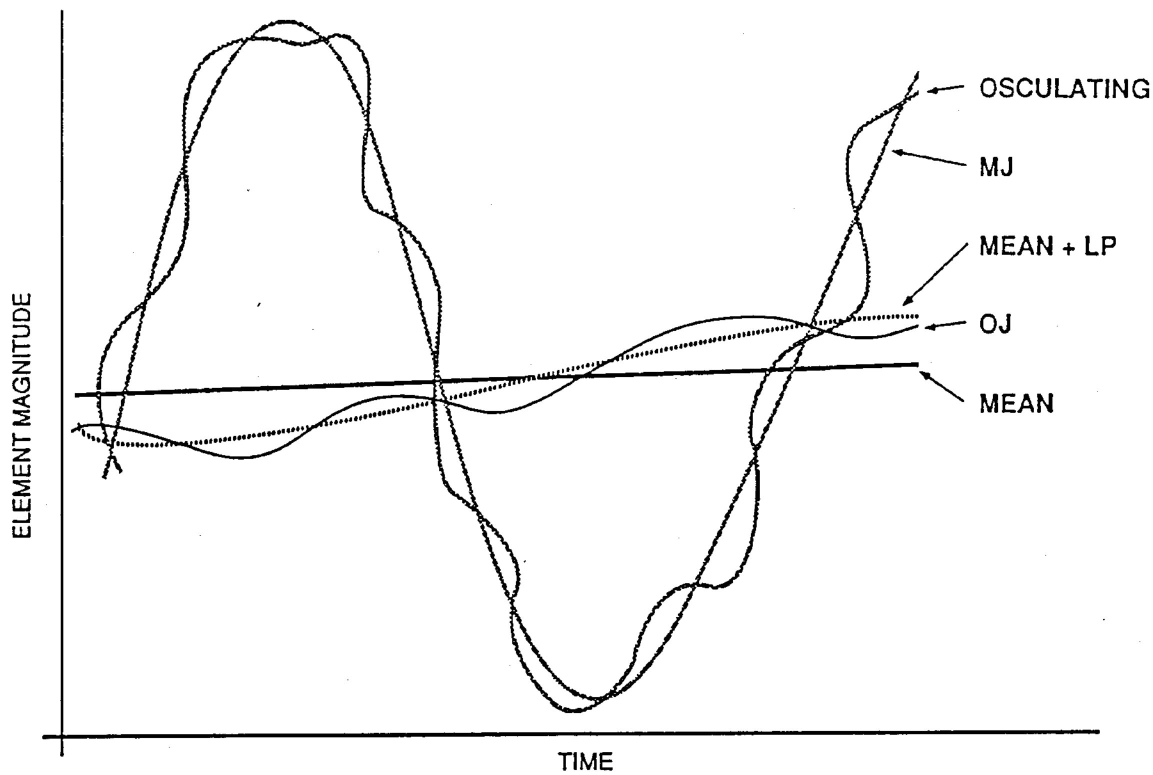

Figure 1. Pictorial Representation of the Relationship between Elements

central body are small so that the distinction between Mean and M+LP is insignificant. However, as discussed below, in high accuracy situations they need to be considered. Other sources such as luni-solar can have large long-period effects. Their presence causes a strong epoch dependence when averaging Osculating Elements over a short time span.

Mean Plus J_2 (MJ): These elements, which are calculated in a manner similar to the calculation of the OJ elements above, are the Mean Elements with the short-period effects of J_2 added. Again, the method of Kozai is assumed. These elements can be used as a first approximation to Osculating Elements.

Intermediate Elements: If any of the short-period effects due to the infinite number of other effects are calculated, they can be either subtracted from the OJ or added to the MJ elements respectively to come closer to doing a true Mean/Osculating conversion. For example, the elements discussed below which define the TRGT are Osculating if only the Earth's gravity field was present but because of the presence of the Sun and Moon are actually Intermediate Elements.

Averaged Elements: These elements are the numerical average of the Osculating Elements (or OJ elements or some type of Intermediate Elements). The Averaged Elements will be equal to the Mean Elements associated with the original Osculating elements to the accuracy of the averaging technique. The only exception to this procedure is the case of small eccentricity, that is when the Mean e is less than the short period effect on it from J_2. This is because any value of e cannot be less than zero. This problem is easily overcome by averaging the OJ value of e rather than the Osculating value.

Estimated Elements: These elements are derived from empirical observations. They will be the osculating values plus the observational errors and the systematic errors in the modeling process.

Method of Investigation

The first step in designing a repeat ground track orbit is to calculate the Mean Elements which give the desired number of orbits in a time period close to the nominal value. For a near circular satellite the ground track repeat pattern will be predominantly determined by the semi-major axis (a) and the inclination (i). The former defines the period while i is a factor in the nodal precession rate due to the zonal harmonics. For the TOPEX/POSEIDON mission, two particular locations on the Earth's surface have to be overflown. This reduces the number of choices for a and i combinations to a reasonable amount.

For this mission it was also decided to choose the eccentricity (e) and argument of perigee (ω) so that their time derivatives were close to zero. This type of "Frozen Orbit" was chosen to minimize the effect of variations in the eccentricity and argument of perigee on the ground track. The design values of a and i for the TOPEX/POSEIDON can be solved for analytically when only the J_2 zonal is used in the propagation. When higher order zonals are used to propagate the Mean Elements the process of determining a and i is one of trial and error though the use of an automatic search technique is possible.

The number of zonals included in the propagation can be increased until the addition of higher degree terms causes a negligible change in the orbit (J_{17} is satisfactory for TOPEX/POSEIDON). Note that when the equations of motion are averaged, additional higher order correction terms are introduced. For example, the value of J_2^2 for the Earth is the same magnitude as J_4 and thus the second order oblateness term needs to be included in the propagation (Ref.3). The proper J_2^2 terms (Ref. 4) have been implemented into the long-term propagator module of the software (Ref. 5) used in this study. However, it should be remembered that this complication is not present when using the non-averaged equations.

The above process of including enough zonal terms to account for the secular effects results in a ground track which is useful for most applications. However, when higher accuracy is required, the next step is to propagate with some of the higher degree and order terms so that the resulting ground track more closely resembles the true ground track defined by the Osculating Elements. The actual propagation can be easily done (albeit more slowly) with some sort of Cowell type integration which inherently includes all the short period terms due to the geopotential chosen. However, the proper choice of the initial conditions for these Intermediate Elements is a separate problem. In order to derive the proper starting values from the predetermined Mean values, the short period effects of the geopotential must be known at that epoch. Many people have done this for the J_2 term only (Ref. 1, 2) and some work has been done for the higher degree terms. However, no analytical method is available for the tesseral terms.

The procedure to determine the initial conditions for propagating with tesserals follows in this manner. A first approximation is obtained by adding the analytical representation of the short period effects due to J_2 to the Mean Elements to give the MJ Elements. The MJ elements are then propagated with the desired geopotential. Each element will then have a signal which includes short period terms. The average value can then be determined by plotting or more accurately by a numerical integrator. These average values will differ by a small amount from the original Mean Elements. The differences are the amounts which the initial conditions have to be adjusted so that the average of their propagated values is equal to the Mean values.

This method has been applied for the design of the TOPEX/POSEIDON operational orbit. The fact that the tesseral effects should be considered when meeting the ground track repeat and overflight criteria was an important result.

Results

Preliminary studies were aimed at obtaining an idea of the size of the effect of the tesseral terms on the orbital elements and ground track. This was done by comparing the results of a Cowell type integration with a full 13X13 (GEM10B) gravity field with the 13x0 field containing only zonal terms. The actual TOPEX/POSEIDON orbit design used a 17x17 field and is also discussed below. The effect of the tesserals on the semi-major axis can be determined by differencing the values that result from propagating with the two gravity fields. (See Figure 2). Note that the approximate 30 m offset in the average of the curve is due to the improper initial conditions used when propagating with the tesserals. If this difference in Mean semi-major axis is not adjusted for, a secular run-off in the ground track will result.

Figure 3 represents the difference in the equator crossings for the two gravity fields after the initial conditions have been adjusted. The maximum ± 150 m difference in the equator crossings is significant compared to the requirement that the equator crossings be maintained within a 2 Km band at the equator. However, as discussed below, the offset repeats for each 10-day orbit cycle. Therefore, since the values form the tesseral propagation are closer to the actual ground track of the satellite it is better to center the 2 Km band about them. Following the definitions in Reference 6, the ground track which corresponds to the equally spaced equator crossings and the zonal gravity field is named the Mean Reference Ground Track (MRGT). The ground track corresponding to the full Earth gravity field is called the Target Reference Ground Track (TRGT). It has been recommended to the Navigation Team to design the orbit maintenance maneuvers (which are necessary because of the luni-solar and drag perturbations) to keep the ground track within ± 1 Km of the TRGT.

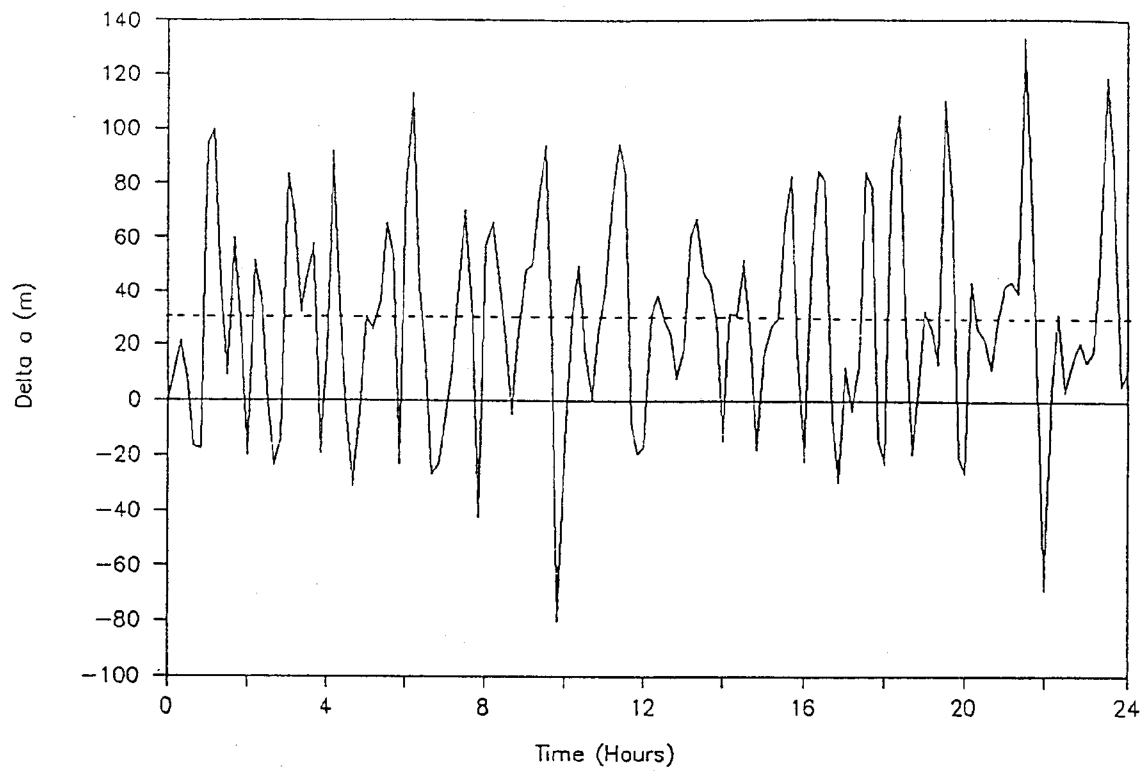

Figure 2. Delta Semi-Major Axis (13x13-13x0)

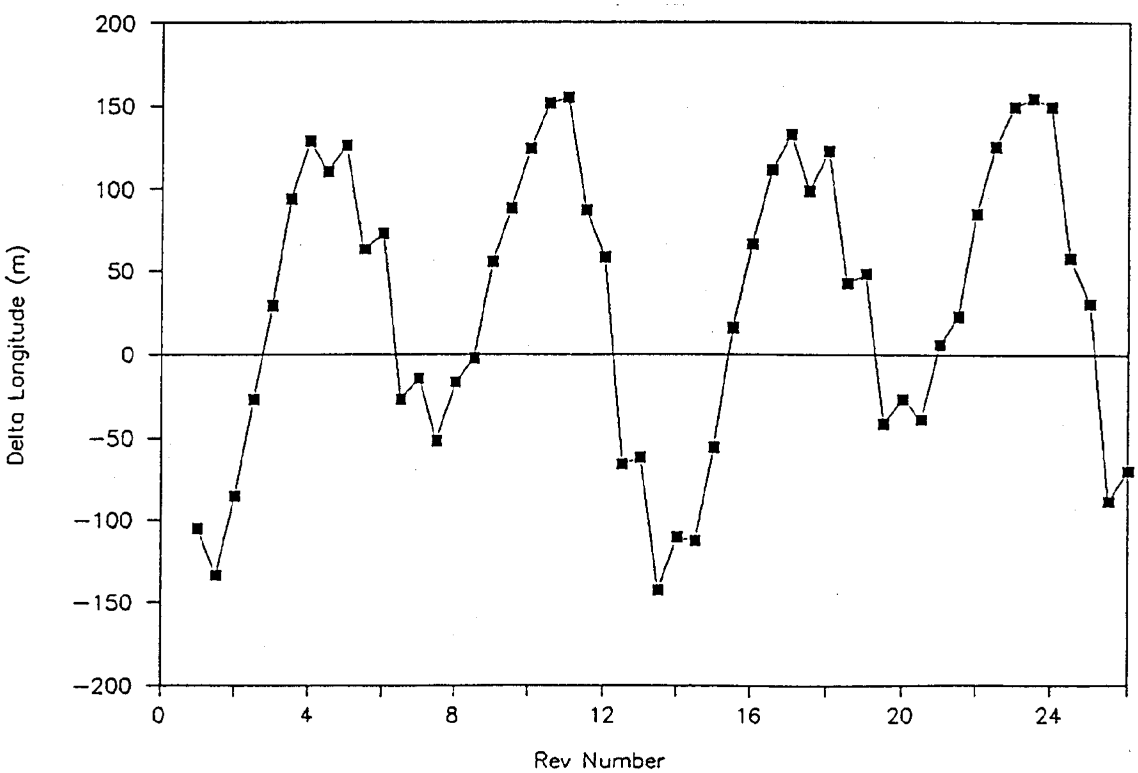

Figure 3. Delta Equator Crossings (13x13 - 13x0)

Although the equator crossings may be used as the gauge of ground track maintenance, the actual intent of the mission is to fly within ± 1 Km of the reference (TRGT) throughout the orbit. Two areas of particular interest are the NASA and CNES verification sites where in-situ measurements will be used to calibrate the altimeters on board the TOPEX/POSEIDON satellite. Both sites are close to 35° N latitude, so the difference between the MRGT and TRGT at this latitude was investigated. Figure 4 shows this difference for all crossings at this latitude. It should be pointed out that because of the northeasterly direction of the ground track, a 200 m longitudinal offset actually corresponds to a closest approach offset of only 179 m. For the actual longitudes of the verification sites, the orbit can be adjusted (see below) so the offsets are both below 100 m which is within the allowance made in the exact definition of the verification point.

During the orbit design for the TOPEX/POSEIDON mission, the Mean Elements can be chosen to give the repeat equator crossing after 127 revs and overflight of the verification sites to any accuracy within the limitations of the gravity field and other constants. In this case this was done to less than 10 m.

The method of choosing initial conditions of the Intermediate Elements when using the full 17x17 field is described above. A more direct method of obtaining the same result is to adjust the initial conditions so that the repeat and overflight criteria are obtained. So far only the semi-major axis, inclination and longitude of ascending node have been adjusted beyond the J_2 conversion. There is an additional smaller effect on the ground track when the other three Keplerian elements are further adjusted (see below). In the present TRGT, the equator crossing after 10 days again repeats to less than 10 m. There is somewhat more variability (up to 100 m) in the verification site overflights. This will be further reduced when the initial conditions of the other in-plane elements are adjusted. This is substantiated in a separate study (in progress) which indicates a significant sensitivity of the ground track to offsets in the values of eccentricity and argument of perigee.

Conclusions

The 17^{th} order terms of a gravity field can be thought of as a sinusoidal variation of 17 cycles around the circumference of the body. This represents a scale of over 10° or 1000 Km. Thus, if the ground track is kept within a 2 Km band, the satellite essentially experiences the same Earth gravity perturbations on each cycle. This results in s hort periodic effects that repeat. Numerically, this was confirmed as shown in Figure 5 where the longitude crossings are compared from one orbit cycle to

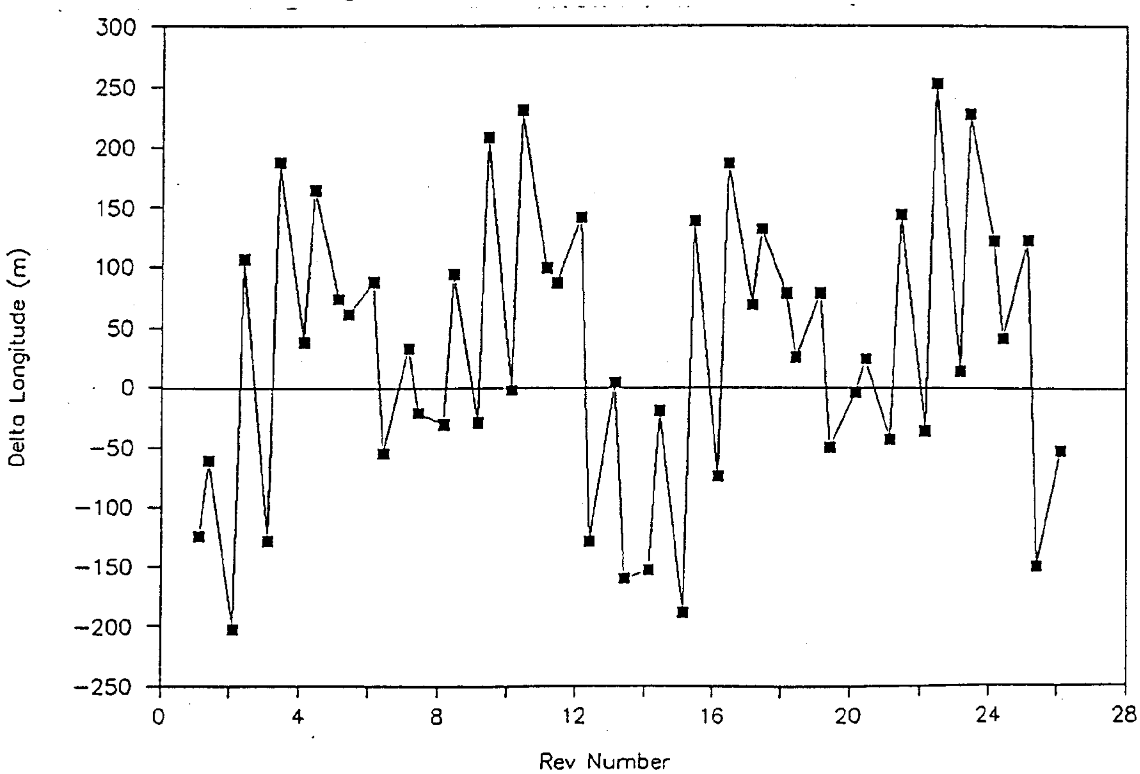

Figure 4. Delta Crossings at 3° Latitude (13x13 - 13x0)

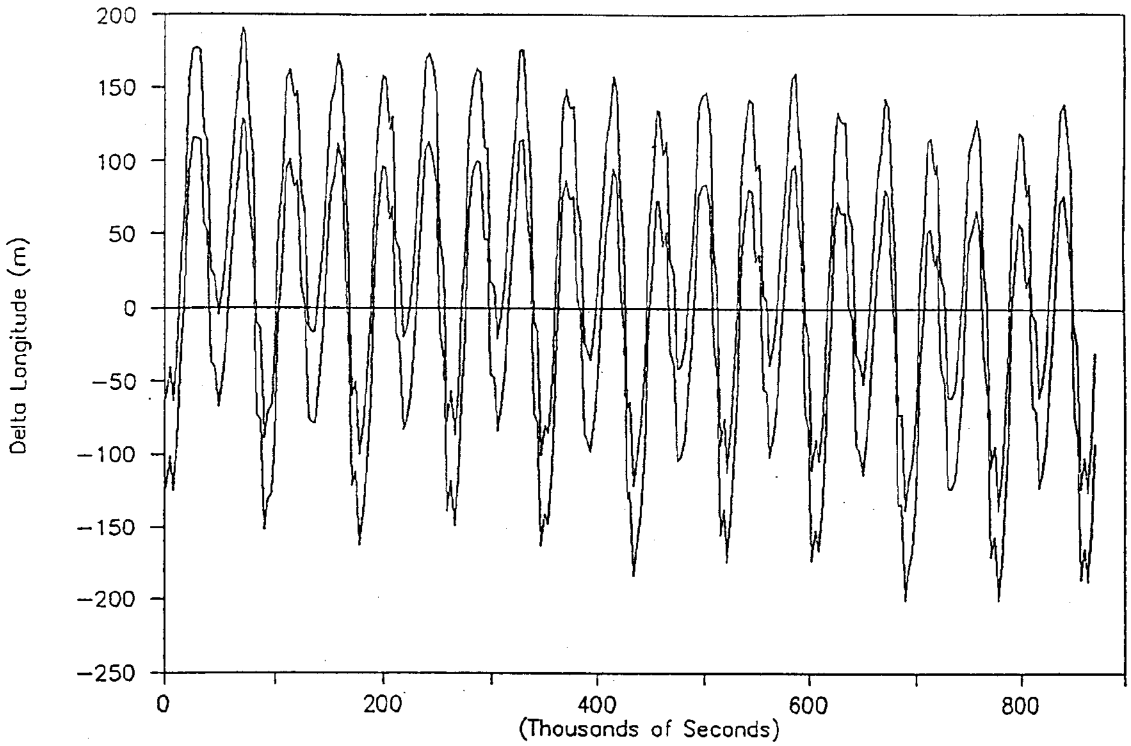

Figure 5. First and Second 10 day Cycle Equator Crossings (17x17 - 2x0)

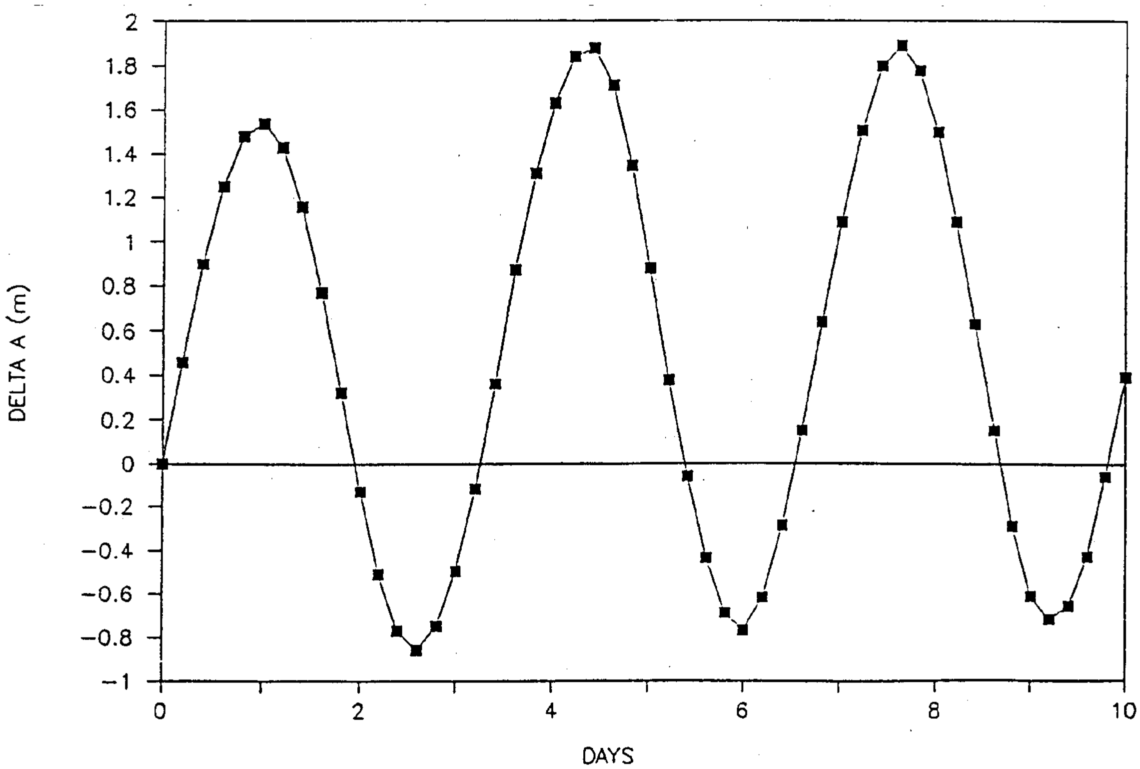

Figure 6. Resonance Effects on Semi-major Axis (17x17+J25 - 17x0+J25)

the next. In this figure, all the secular effect has not been completely removed by adjusting the initial conditions, however this makes the comparison of the short periodic effects between the different cycles easier to compare.

There are some other, albeit smaller, medium period effects both of the resonant and non-resonant type. To study these effects a comparison of the same gravity fields described above was done, however in this case, the long-term propagator which includes secular, long and medium period effects was used. Thus the short period terms did not show up in the difference. The resonant effect on the semi-major axis is shown in Figure 6. The period of this perturbation appears to be exactly one-third of the cycle length of 9.9 days. The physical cause of this 3:1 resonance would require further investigation, though the (3,1) gravity field term is known to be relatively important to TOPEX/POSEIDON orbit (Ref. 7). However, the same principle of the repeating orbit experiencing the same Earth gravity effects is still valid.

In summary, when a satellite is given the proper initial conditions corresponding to Mean Elements that repeat under the central body gravity field, then the perturbations from all that gravity field will also repeat. Further, in the presence of other non-commensurate force perturbations which are accounted for by maneuvers so that the ground track remains close to the target, then again the central body forces are repetitive.

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